Math 42 Second Exam — February 20, 2014

Name: _____ **SUID#:** _____

Circle your section:			
Junsoo Ha	Nick Ronchetti	Li-Cheng Tsai	Sunny Ahuja
03 (10-10:50am)	05 (11-11:50am)	02 (10-10:50am)	ACE
08 (1:15-2:05pm)	07 (9-9:50am)	04 (11-11:50am)	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of the question page or other extra space provided at the front of this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, March 6**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: _____

1. (12 points)

(a) Evaluate $\int_0^\infty xe^{-x^2}\,dx$ or explain why its value does not exist; show all reasoning.

(b) Determine whether $\int_0^1 \frac{\ln(1+x)}{x^2}dx$ converges or diverges; give complete reasoning.

2. (10 points) Determine, with justification, whether each series converges. *If the series converges, find its sum.*

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n-1} - 5^{n+2}}{3^{2n}}$

Problem instructions, repeated: Determine, with justification, whether each series converges. *If the series converges, find its sum.*

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

3. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^{2/3}}{n^{3/2} - n + 1}$

Problem instructions, repeated: Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\, n!}{(2n-1)!}$

4. (10 points) One of three possibilities holds for each of the series below: (i) it converges absolutely, (ii) it converges but does not converge absolutely, or (iii) it diverges. Determine which possibility holds for each of the series below; indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n \ln n}{2n}$

Problem instructions, repeated: One of three possibilities holds for each of the series below: (i) it converges absolutely, (ii) it converges but does not converge absolutely, or (iii) it diverges. Determine which possibility holds for each of the series below; indicate clearly which tests you use and how you apply them.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{2 + (-1)^n}$

5. (12 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(3-2x)^n}{(n^3+2)\,5^n}$$

6. (12 points) Match each function below with its power series, listed among the choices below. You do not need to justify your answers. (Not all of the series have a match, but every function has a match.)

(A) $-x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \frac{x^8}{4} - \cdots$

(B) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots$

(C) $1 - \frac{2^2}{2!}x^2 + \frac{2^4}{4!}x^4 - \frac{2^6}{6!}x^6 + \cdots$

(D) $-x^3 - \frac{x^5}{2} - \frac{x^7}{3} - \frac{x^9}{4} - \cdots$

(E) $\frac{1}{10} + \frac{x}{100} + \frac{x^2}{1000} + \frac{x^3}{10000} + \cdots$

(F) $1 - x^2 + x^4 - x^6 + \cdots$

(G) $x - x^3 + x^5 - x^7 + \cdots$

(H) $x^3 - \frac{x^5}{2} + \frac{x^7}{3} - \frac{x^9}{4} + \cdots$

(I) $2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \cdots$

(J) $\frac{1}{10} - \frac{x}{100} + \frac{x^2}{1000} - \frac{x^3}{10000} + \cdots$

(K) $1 + 2x + \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 + \cdots$

(L) $1 - 2x + 3x^2 - 4x^3 + \cdots$

Function	Series (choose one of (A) through (L))
$\cos(2x)$	
$x\ln(1+x^2)$	
e^{2x}	
$\frac{1}{10+x}$	
$\frac{1}{1+x^2}$	
$\ln(1-x^2)$	

7. (12 points) Let $f(x) = x^{5/4}$.

   (a) Find $T_2(x)$, the degree-2 Taylor polynomial for f centered at 16. (Note: $16^{1/4} = 2$, so $16^{5/4} = 32$.)

   (b) Use T_2 to obtain an approximation for $17^{5/4}$.

(problem continued from previous page)

(c) Determine the accuracy of your approximation from part (b), explaining the steps of your reasoning, and giving your answer in sentence form.

(d) Find a set of values of x for which $x^{5/4} \approx T_2(x)$ with an error of no more than ± 0.01. (*Hint:* the set should be an interval of the form $[16 - d, 16 + d]$.)