Math 42 First Exam — January 28, 2014

Name: ____ **SUID#:** ______________

Circle your section:			
Junsoo Ha	Nick Ronchetti	Li-Cheng Tsai	Sunny Ahuja
03 (10-10:50am)	05 (11-11:50am)	02 (10-10:50am)	ACE
08 (1:15-2:05pm)	07 (9-9:50am)	04 (11-11:50am)	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 9 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of the question page or other extra space provided at the front of this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, February 11**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

1. (12 points) Compute each of the following integrals, showing all of your reasoning.

   (a) $\displaystyle\int_0^2 t^2 e^t \, dt$

   (b) $\displaystyle\int \sec^4 2\theta \tan^4 2\theta \, d\theta$

2. (13 points) Compute each of the following integrals, showing all of your reasoning.

(a) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

(b) $\displaystyle\int_{-1}^{1} \frac{dx}{e^x+e^{-x}}$

3. (8 points) Show all your steps in computing $\displaystyle\int \frac{x^6 - 9}{x^4 - x^2}\, dx$

4. (8 points) Let $f(x) = e^{1/x}$. In this problem, we study approximations of the following integral:

$$\int_1^3 e^{1/x}\,dx \qquad \text{(Note: don't evaluate this integral.)}$$

(a) Show that if this integral is approximated using the Midpoint Rule with $n = 10$ subintervals, then the approximation is accurate to within 0.1 units. Explain all reasoning; you may make use of the fact that $f''(x) = \dfrac{e^{1/x}(1+2x)}{x^4}$.

(b) How many subintervals n would ensure that the Midpoint Rule approximation is accurate to within 10^{-5} units? Give a valid n in simplified form. (As long as you justify your answer, you do not have to find the best possible value.)

5. (9 points) An *ice core* is a long, thin cylinder of solid ice removed from a glacier by a vertical drilling process. One such core taken from an Antarctic ice sheet is 1200 cm in length; it is studied to determine the varying properties of glacier ice whose depth, x, ranges from 0 cm (i.e., the surface) to 1200 cm. For each such x, a thin cross section of the core is circular of constant radius 10 cm, and has mass density $d(x)$ g/cm^3, according to the values given in the following table:

x	0	200	400	600	800	1000	1200
$d(x)$	0.71	0.87	0.89	0.905	0.912	0.916	0.917

(a) Set up an integral involving the function d which expresses the total mass of the ice core in grams.

(b) Write an expression, involving only numbers, that approximates your integral from part (a) by the Trapezoidal Rule, using all of the data in the table. Do not simplify your answer.

(c) Write an expression, involving only numbers, that approximates your integral from part (a) by Simpson's Rule, using all of the data in the table. Do not simplify your answer.

6. (8 points) The city of Centropolis has the shape of a circle 8 miles in radius; the population density at a point r miles from the city center is given by $d(r) = \frac{10^5}{\pi} e^{-r^2/2}$ people per square mile.

   (a) Set up, but do not evaluate, an integral that gives the total population of the city.

   (b) Find the distance R with the property that 20,000 people live within R miles of the city center. (Your final expression for R should not involve integral symbols.)

7. (8 points) Let R be the bounded region in the xy-plane enclosed by the curves $y = 5 + 2x - x^2$ and $y = 1 - x$.

   (a) Set up, but do not evaluate, an integral representing the area of R. As justification, draw a picture with a sample slice labeled.

   (b) Suppose a V is a three-dimensional solid whose base is R, and whose cross-sections perpendicular to the x-axis are regular hexagons. Set up, but do not evaluate, an integral that gives the volume of V. (Note: the area of a regular hexagon of side length s is $\frac{3}{2}s^2\sqrt{3}$.)

8. (12 points) Let R be the bounded region in the *first quadrant* of the xy-plane enclosed by the curves $y = x^3$ and $y = 4x$.

   (a) Set up (but do not evaluate) two distinct integrals, each in terms of a single variable, which represent the volume of the solid obtained by rotating R about the y-axis. Justify your answer by drawing pictures, labeling sample slices, and citing the methods used.

For easy reference, R is the bounded region in the *first quadrant* of the xy-plane enclosed by the curves $y = x^3$ and $y = 4x$.

(b) Set up (but do not evaluate) two distinct integrals, each in terms of a single variable, which represent the volume of the solid obtained by rotating R about the line $y = -1$. Justify your answer by drawing pictures, labeling sample slices, and citing the methods used.