Math 42 Second Exam — February 21, 2013

Name: ____ **SUID#:** ______________

Circle your section:				
Kenji Kozai 02 (10-10:50am) 07 (1:15-2:05pm)	Jeremy Leach 04 (11-11:50am) 09 (2:15-3:05pm)	Xiaodong Li 05 (11-11:50am) 10 (2:15-3:05pm)	Mike Lipnowski 03 (10-10:50am) 08 (1:15-2:05pm)	Nisan Stiennon ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of a page or one of the extra sheets provided in this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, March 7**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	10	10	10	15	13	12	12	12	94
Score:									

1. (10 points) Determine, with justification, whether each series converges. *If the series converges, find its sum.*

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4^n}{(-5)^{n-1}} - \frac{1}{2^n} \right)$

2. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+4}{5n^3-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{(n+1)!}$

3. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} 2ne^{-n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{2\cos n}{n^4 - 1}$

4. (15 points) (Two pages)

   (a) Show that the series $s = \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3}$ converges.

   (b) We can approximate s by computing the tenth partial sum of the series, which is found to be

$$s_{10} = 0.8325298\ldots$$

   Express the remainder (error) $R_{10} = s - s_{10}$ as a series of its own, and explain why R_{10} is positive.

Facts from prev. page: $s = \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3}$ $s_{10} = 0.8325298\ldots$ $R_{10} = s - s_{10}$ (positive, by (b))

(c) Show that $R_{10} \leq \frac{1}{200}$. (Hint: Use the Comparison Test to relate R_{10} to a series that can be analyzed with the Integral Test.)

(d) Put the information in parts (b) and (c) together to write a statement of the form

$$A \leq \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3} \leq B$$

for appropriate values A and B. (A and B should be expressed as numbers, not as infinite summations or unevaluated integrals, but do not have to be in simplified form.)

(e) Use (d) to give an improved approximation for s. In a mathematically precise statement, express the error in this new approximation.

5. (13 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x+3)^n}{\sqrt{n+1}}$$

6. (12 points) Suppose the power series $\sum_{n=0}^{\infty} c_n x^n$ converges for $x = 3$, but diverges for $x = -5$; no other information about the values of c_n is given. Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

(a) $\sum_{n=0}^{\infty} 4^n c_n$ Converges Diverges Inconclusive

(b) $\sum_{n=0}^{\infty} (-2)^n c_n$ Converges Diverges Inconclusive

(c) $\sum_{n=1}^{\infty} n3^n c_n$ Converges Diverges Inconclusive

(d) $\sum_{n=0}^{\infty} \frac{6^n c_n}{n+1}$ Converges Diverges Inconclusive

(e) $\sum_{n=0}^{\infty} (c_n)^2$ Converges Diverges Inconclusive

(f) $\sum_{n=0}^{\infty} n! c_n$ Converges Diverges Inconclusive

7. (12 points) Determine, showing all reasoning, a power series centered at 0 for each of the functions given below, and give the *radius of convergence*.

   (a) $g(x) = \dfrac{x}{4 + x^2}$

   (b) $h(x) = \dfrac{1}{(1 - x)^2}$

8. (12 points) Let $f(x) = \cos x$.

(a) Find $T_4(x)$, the degree-4 Taylor polynomial for f centered at $\frac{\pi}{4}$. $\left(\text{Hint: } \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}.\right)$ Show all steps of your reasoning.

(b) Use T_4 to obtain an approximation for $\cos\frac{\pi}{6}$. Give your approximation as an expression involving only numbers, not unevaluated trig functions; but you do not need to put it in simplified form.

(c) Determine the accuracy of your approximation from part (b), explaining all your reasoning, and giving your final conclusion in sentence form. (Again, use only numbers, not unevaluated trig functions, but you don't need a simplified expression.)