Math 42 First Exam — January 31, 2013

Name: ____ **SUID#:** ______________

Circle your section:				
Kenji Kozai 02 (10-10:50am) 07 (1:15-2:05pm)	Jeremy Leach 04 (11-11:50am) 09 (2:15-3:05pm)	Xiaodong Li 05 (11-11:50am) 10 (2:15-3:05pm)	Mike Lipnowski 03 (10-10:50am) 08 (1:15-2:05pm)	Nisan Stiennon ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 10 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of a page or one of the extra sheets provided in this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, February 14**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	9	**Total**
Points:	12	13	8	5	12	9	12	13	10	94
Score:										

1. (12 points) Evaluate each of the following, or explain why its value does not exist; show all reasoning.

(a) $\displaystyle\int_1^e (\ln x)^2 \, dx$

(b) $\displaystyle\int_{-1}^1 \frac{e^x}{e^x - 1} \, dx$

2. (13 points) Compute each of the following integrals, showing all of your reasoning.

   (a) $\displaystyle\int \sec^3 x \tan x \, dx$

   (b) $\displaystyle\int \frac{dt}{t^2\sqrt{9-t^2}}$

3. (8 points) Show all your steps in computing $\displaystyle\int \frac{x^6+16}{x^6-16x^2}\,dx$

4. (5 points) *Set up* the complete partial fraction decomposition of the following rational expression, in terms of undetermined constants A, B, etc., as appropriate. (*Do not* attempt to clear fractions or determine the values of any of the constants!)

$$\frac{x^8 + x^4 + 1}{(x+2)^2(x-3)(x^2+1)^2(x^2+4x+5)} =$$

5. (12 points)

(a) Evaluate $\displaystyle\int_e^\infty \frac{1}{x\,(\ln x)^2}\,dx$ or explain why its value does not exist; show all reasoning.

(b) Determine whether $\displaystyle\int_0^\pi \frac{\cos^2 x + 1}{x^{2/3}}\,dx$ converges or diverges; give complete reasoning.

6. (9 points) Put the following quantities in increasing order (from smallest number to largest). You do not need to justify your answer.

(A) $\int_2^6 e^{-t}\,dt$

(B) $e^{-2}+e^{-3}+e^{-4}+e^{-5}$

(C) $e^{-3}+e^{-4}+e^{-5}+e^{-6}$

(D) $e^{-2.5}+e^{-3.5}+e^{-4.5}+e^{-5.5}$

(E) The number 0

(F) $\frac{1}{2}\left(e^{-2}+2e^{-3}+2e^{-4}+2e^{-5}+e^{-6}\right)$

(G) $\frac{1}{2}\left(e^{-2.25}+e^{-2.75}+e^{-3.25}+e^{-3.75}+e^{-4.25}+e^{-4.75}+e^{-5.25}+e^{-5.75}\right)$

(H) $\frac{1}{4}\left(e^{-2}+2e^{-2.5}+2e^{-3}+2e^{-3.5}+2e^{-4}+2e^{-4.5}+2e^{-5}+2e^{-5.5}+e^{-6}\right)$

(I) $e^{-6}-e^{-2}$

7. (12 points) Let $f(x) = \frac{1}{x} - \sin x$. In this problem, we study approximations of the following integral:

$$\int_1^5 \left(\frac{1}{x} - \sin x\right)\, dx \qquad \text{(Note: there is no need to evaluate integral)}$$

(a) Write an expression involving only numbers (and the sine function) that approximates the above integral using Simpson's Rule with 4 subintervals. You do *not* have to simplify this expression.

(b) How accurately does your expression of part (a) approximate the above integral? *State your final answer, involving rational numbers, in a complete sentence.* Show all reasoning.

(c) Find a value of n which guarantees that a Simpson's Rule approximation of the above integral using n subintervals is accurate to within 10^{-8}. Your final answer should give a valid n in simplified form, and be fully justified, but it need not be optimal in any sense.

8. (13 points) Consider the region A in the xy-plane bounded by the curves $x = 0$, $y = e^x$, and $y = ex$.

   (a) Set up, but do not evaluate, an integral in terms of a single variable that represents the area of A. Justify your answer by sketching a picture and labeling a sample slice.

   (b) Set up, but do not evaluate, an integral that represents the volume of the solid obtained by rotating A about the y-axis. Justify your answer by citing the method used, sketching a picture and labeling a sample slice.

Quick reference: as before, A is the region bounded by the curves $x = 0$, $y = e^x$, and $y = ex$.

(c) Set up, but do not evaluate, an integral that represents the volume of the solid obtained by rotating A about the line $y = -1$. Justify your answer by citing the method used, sketching a picture and labeling a sample slice.

9. (10 points) Consider the region R in the xy-plane bounded by the curves $x = y^2$ and $x = 4 - y^2$.

(a) Set up, but do not evaluate, an integral in terms of a single variable that represents the volume of the solid obtained by rotating R about the line $y = 2$. Justify your answer by drawing a picture of R and labeling a sample slice.

(b) Suppose a three-dimensional solid V has the following properties: it has R as its base; and cross-sections of V perpendicular to the y-axis are squares. Set up, but do not evaluate, an integral that gives the volume of V.