Math 42 Second Exam — February 28, 2012

Name: ______________________________ **SUID#:** ______________

Circle your section:				
Megan Bernstein	Sukhada Fadnavis	Frederick Tsz-Ho Fong	Nisan Stiennon	Tracy Nance
04 (11-11:50am)	02 (10-10:50am)	08 (1:15-2:05pm)	03 (10-10:50am)	ACE
07 (1:15-2:05pm)	05 (11-11:50am)	09 (2:15-3:05pm)	10 (2:15-3:05pm)	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

- Please check that your copy of this exam contains 9 numbered pages and is correctly stapled.

- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.

- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.

- If you need extra room for your answers, use the back sides of each page. If you must use extra paper, use only that provided by teaching staff; make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, March 13**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	10	10	12	10	13	12	11	10	88
Score:									

1. (10 points) For this problem, use the following information:
   - If g is a normal ("bell-shaped" or "Gaussian") probability density function, then g has the general form
   $$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$
   - A partial list of approximate values of the function
   $$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt \quad \text{is given at right:}$$

$F(0.25) \approx 0.60$	$F(1.75) \approx 0.960$
$F(0.5) \approx 0.69$	$F(2.0) \approx 0.977$
$F(0.75) \approx 0.77$	$F(2.25) \approx 0.988$
$F(1.0) \approx 0.84$	$F(2.5) \approx 0.994$
$F(1.25) \approx 0.89$	$F(2.75) \approx 0.997$
$F(1.5) \approx 0.93$	$F(3.0) \approx 0.999$

Suppose that a manufacturer of voltmeters tests its devices for quality control before shipment, and discovers that the amount of imprecision in a randomly selected voltmeter is approximately normally distributed with mean 0.25 mV and standard deviation 0.5 mV. (Note that by "imprecision" of a device we mean the difference between a "true" voltage and the device's measurement of that voltage; this difference can be either positive or negative.)

(a) Based on the above information, what is the probability that a randomly chosen voltmeter has a positive value of imprecision? Your answer should be a number; justify it by writing an integral expression that represents this probability and showing how to find its value.

(b) If the imprecision of a voltmeter is less than 1.0 mV in absolute value, then it is shipped out. Otherwise, it is sent back to be recalibrated. Approximately what fraction of the voltmeters are sent back for recalibration? (Again use an integral expression as part of your justification.)

2. (10 points) Determine, with justification, whether each series converges. *If the series converges, find its sum.*

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{n-1}}{3^{2n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{2}{n}\right)$

3. (12 points) In each of the following parts, give a formula for a_n so that the series $\sum_{n=1}^{\infty} a_n$ has the specified property or properties, or state that such a series cannot exist. *You do not need to justify your answers.* (Please treat each question as independent from the others; properties do not carry over from part (a) to part (b), etc.)

(a) The series $\sum_{n=1}^{\infty} a_n$ diverges and $\lim_{n\to\infty} a_n = 0$.

(b) The series $\sum_{n=1}^{\infty} a_n$ converges and $a_n < a_{n+1}$ for all $n \geq 1$.

(c) The series $\sum_{n=1}^{\infty} a_n$ converges and $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$.

(d) The series $\sum_{n=1}^{\infty} a_n$ converges absolutely and the series $\sum_{n=1}^{\infty} (a_n)^2$ diverges.

4. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3-2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n\, n!(n+1)!}$

5. (13 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(3-x)^{3n}}{3^{3n}(\ln n)}$$

6. (12 points) Suppose the power series $\sum_{n=0}^{\infty} c_n(x-2)^n$ converges for $x = 7$ but not for $x = -4$; no other information about the values of c_n is given. Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

(a) $\sum_{n=0}^{\infty} 2^n c_n$ Converges Diverges Inconclusive

(b) $\sum_{n=0}^{\infty} 7^n c_n$ Converges Diverges Inconclusive

(c) $\sum_{n=1}^{\infty} n c_n$ Converges Diverges Inconclusive

(d) $\sum_{n=0}^{\infty} 6^n \frac{c_n}{n+1}$ Converges Diverges Inconclusive

(e) $\sum_{n=0}^{\infty} |c_n|$ Converges Diverges Inconclusive

(f) $\sum_{n=1}^{\infty} \frac{1}{n+(c_n)^4}$ Converges Diverges Inconclusive

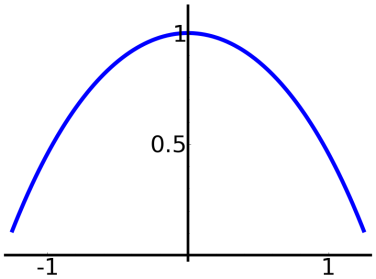
7. (11 points) Let $f(x) = x^{1/3}$.

(a) Find $T_3(x)$, the degree-3 Taylor polynomial for f centered at 8.

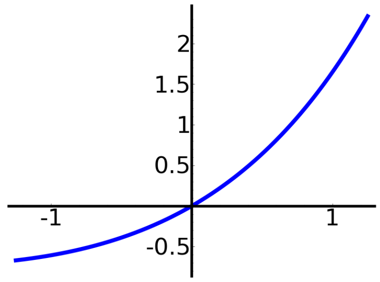
(b) Use T_3 to obtain an approximation for the cube root of 7.9. (You do not need to simplify your answer.)

(c) Determine the accuracy of your approximation from part (b), explaining the steps of your reasoning, and giving your final conclusion in sentence form.

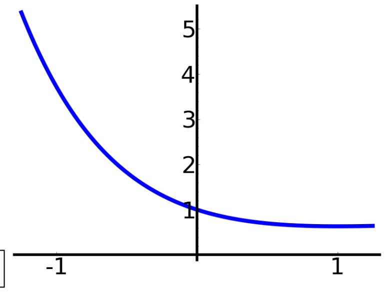
8. (10 points) Match each power series below to its graph, chosen from among the six displayed. (Note that each series has a match, but exactly one of the graphs does not correspond to any power series in the list). You do not need to justify your answers.



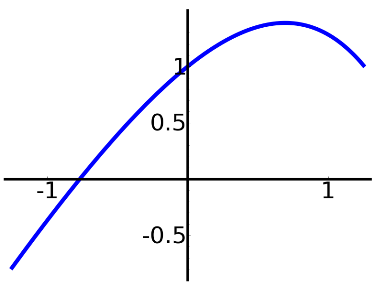
I



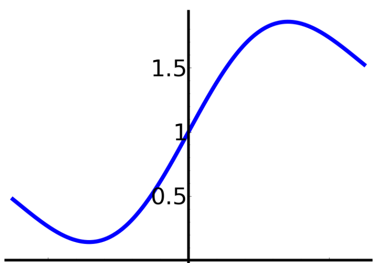
II



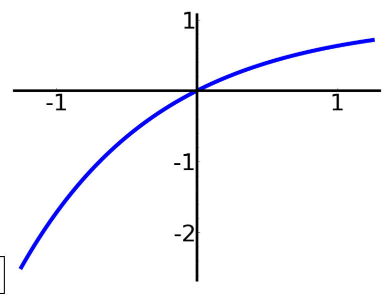
III



IV



V



VI

Series	I, II, III, IV, V, or VI
$f(x) = 1 + 2x - 2x^3 + \dfrac{2x^5}{2!} - \dfrac{2x^7}{3!} + \cdots$	
$f(x) = x - \dfrac{x^2}{2!} + \dfrac{x^3}{3!} - \dfrac{x^4}{4!} + \dfrac{x^5}{5!} - \cdots$	
$f(x) = 1 - \dfrac{x^2}{2!} - \dfrac{x^4}{4!} - \dfrac{x^6}{6!} - \dfrac{x^8}{8!} - \cdots$	
$f(x) = 1 + x - \dfrac{x^2}{2!} - \dfrac{x^3}{3!} - \dfrac{x^4}{4!} - \cdots$	
$f(x) = x + \dfrac{x^2}{2} + \dfrac{x^3}{2^2 \cdot 2!} + \dfrac{x^4}{2^3 \cdot 3!} + \dfrac{x^5}{2^4 \cdot 4!} + \cdots$	