Math 42 First Exam — February 2, 2012

Name: ______________________________ **SUID#:** ____________

Circle your section:				
Megan Bernstein 04 (11-11:50am) 07 (1:15-2:05pm)	Sukhada Fadnavis 02 (10-10:50am) 05 (11-11:50am)	Frederick Tsz-Ho Fong 08 (1:15-2:05pm) 09 (2:15-3:05pm)	Nisan Stiennon 03 (10-10:50am) 10 (2:15-3:05pm)	Tracy Nance ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- Please check that your copy of this exam contains 10 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- If you need extra room for your answers, use the back sides of each page. If you must use extra paper, use only that provided by teaching staff; make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, February 16**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	12	13	7	12	9	14	10	11	88
Score:									

1. (12 points) Evaluate each of the following integrals, showing all of your reasoning.

(a) $\displaystyle\int_0^{\pi/4} \cos^4\theta\, d\theta$

(b) $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\, dx$

2. (13 points) Evaluate each of the following integrals, showing all of your reasoning.

(a) $\int x^2 \arctan x \, dx$

(b) $\int \frac{dx}{(3 - 2x - x^2)^{3/2}}$

3. (7 points) Evaluate $\displaystyle\int \frac{x^4+x}{x^4-1}\,dx,$ showing all reasoning.

4. (12 points)

   (a) Determine whether $\displaystyle\int_1^\infty \frac{2+\cos x}{x\ln x}\,dx$ converges or diverges; give complete reasoning.

   (b) Determine whether $\displaystyle\int_0^1 \frac{\sqrt{\sin x}}{x}\,dx$ converges or diverges; give complete reasoning.

5. (9 points) A molasses tank has exploded, spreading sticky goo across the ground in all directions. The mass density $\rho(z)$ of molasses (measured in kilograms per square meter) at each point on the ground near the tank is assumed to depend only on the distance z (in meters) from the tank.

   (a) Write an integral involving the function ρ which expresses the total mass of all molasses that lies on the ground within a 60-meter radius of the tank.

   (b) The density was measured experimentally at several points on the ground near the tank. The values collected are:

z (m)	0	10	20	30	40	50	60
$\rho(z)$ (kg/m^2)	3	2.8	2.5	2	1.5	0.8	0.2

   Use the Midpoint Rule to estimate the total mass of molasses as expressed by your integral in (a). Use as much of the data in the table above as possible, and do not simplify your answer.

   (c) Use Simpson's Rule to estimate the total mass of molasses as expressed by your integral in (a). Use all the data in the table above, and do not simplify your answer.

6. (14 points) Let $f(x) = e^{-e^x}$. In this problem, we study approximations of the following integral:

$$\int_0^1 e^{-e^x}\,dx$$

(a) Write an algebraic expression involving only numbers that approximates the above integral using the Trapezoidal Rule with 4 subintervals. You do *not* have to simplify this expression.

(b) Compute $f''(x)$, and show that

$$0 \leq f''(x) \leq \frac{2}{3} \quad \text{for all } x \text{ in } [0,1].$$

(c) Using the fact stated in part (b), show that your Trapezoidal Rule approximation in part (a) is accurate to within $\frac{1}{250}$. (You may cite the fact of part (b) even if you did not prove it.) In addition, explain whether the approximation of part (a) gives an overestimate or underestimate of the integral, or whether it is impossible to tell.

(d) Find a value of n which guarantees that a Trapezoidal Rule approximation of the above integral using n subintervals is accurate to within 10^{-10}. Your final answer should give a valid n in simplified form, and be fully justified, but it need not be optimal in any sense. (You may again apply the fact of part (b), even if you did not prove it.)

7. (10 points) Let R be the bounded region enclosed by the curves $y = \sqrt{x}$ and $y = x^{1/3}$ in the first quadrant.

   (a) Set up two distinct integrals, each in terms of a single variable, representing the area of R. For each, justify your answer by drawing a picture and marking a sample slice. Don't evaluate either integral.

   (b) Set up two distinct integrals, each in terms of a single variable, which represent the volume of the solid obtained by rotating R about the line $x = 1$. Justify your answer by drawing pictures, labeling sample slices, and citing the methods used. Don't evaluate either integral.

8. (11 points) Consider the region R in the xy-plane below the curve $y = xe^{-x}$ and above the portion of the x-axis with $0 \le x \le 2$.

   (a) Set up, but do not yet evaluate, an integral in terms of a single variable which represents the volume of the solid of revolution obtained by rotating R about the y-axis. Justify your answer by drawing a picture, labeling a sample slice, and citing the method used.

   (b) Evaluate the integral of part (a), showing all your steps.

(c) Suppose a three-dimensional solid V has the following properties: it has R as its base; and each cross-section of V perpendicular to the x-axis is an isosceles right triangle with hypotenuse along the base. Set up, but do not evaluate, an integral that gives the volume of V.