Math 42 Second Exam — February 17, 2011

Name: ____ **SUID#:** ______________

Circle your section:				
Jack Hall 05 (11-11:50am) 10 (2:15-3:05pm)	Vitaly Katsnelson 04 (11-11:50am) 07 (1:15-2:05pm)	Maks Radziwill 02 (10-10:50am) 09 (2:15-3:05pm)	David Sher 03 (10-10:50am) 08 (1:15-2:05pm)	Tracy Nance ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, March 8**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	10	10	10	10	12	13	13	10	88
Score:									

1. (10 points) In a certain California town, the probability density function for a random day's total rainfall is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ A(x+1)^{-3/2} & \text{if } x \geq 0 \end{cases}$$

where x is measured in millimeters, and A is a positive constant.

(a) Find A, given that f is a probability density function.

(b) What is the median amount of daily rainfall in this town?

2. (10 points)

(a) Determine with justification if the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

converges, and if so, find the sum.

(b) Express $2.0\overline{134} = 2.0134134134134\ldots$ as a ratio of integers.

3. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^{2n}}{n^3\, 5^n}$

(b) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

4. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \sin^2 \frac{\pi}{n}$

5. (12 points) Suppose that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.

Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

(a) $\sum_{n=1}^{\infty} \left(a_n + \frac{1}{n^2} \right)$ Converges Diverges Inconclusive

(b) $\sum_{n=1}^{\infty} (-1)^n a_n$ Converges Diverges Inconclusive

(c) $\sum_{n=1}^{\infty} \frac{1}{1 + a_n^2}$ Converges Diverges Inconclusive

(d) $\sum_{n=1}^{\infty} \frac{|a_n|}{n}$ Converges Diverges Inconclusive

(e) $\sum_{n=1}^{\infty} n^2 \, a_n$ Converges Diverges Inconclusive

(f) $\sum_{n=1}^{\infty} n! \, a_n$ Converges Diverges Inconclusive

6. (13 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}(3x-1)^n$$

7. (13 points) In each of the parts below, show all the steps in your reasoning.

   (a) Write $\dfrac{1}{1+x^2}$ as a power series about 0, and state the interval of convergence. (Hint: use geometric series.)

   (b) Find a power series for $\arctan x$. What is the radius of convergence?

(c) Express the number $\int_0^{0.1} \frac{\arctan x}{x}\, dx$ as a series.

(d) Find, with complete justification, a partial sum of the series in part (c) that approximates the value of the integral to within 10^{-8}. (You do not need to simplify the sum.)

8. (10 points)

(a) Find, showing all your steps, the Taylor series for e^x with center 0.

(b) Use series to find $\lim_{x\to 0} \dfrac{e^{-x^2}-1}{e^x - x - 1}$. (You may take for granted the fact that the Taylor series for e^x converges to e^x.)