Math 42 First Exam — January 27, 2011

Name: ____ **SUID#:** ______________

Circle your section:				
Jack Hall 05 (11-11:50am) 10 (2:15-3:05pm)	Vitaly Katsnelson 04 (11-11:50am) 07 (1:15-2:05pm)	Maks Radziwill 02 (10-10:50am) 09 (2:15-3:05pm)	David Sher 03 (10-10:50am) 08 (1:15-2:05pm)	Tracy Nance ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 9 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, February 10**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	9	**Total**
Points:	12	13	7	12	6	12	10	10	6	88
Score:										

1. (12 points) Evaluate each of the following integrals, showing all of your reasoning.

   (a) $\displaystyle\int_1^e t^3 \ln t \, dt$

   (b) $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^5}$

2. (13 points) Evaluate each of the following integrals, showing all of your reasoning.

(a) $\int \tan^3 x \sec x \, dx$

(b) $\int \frac{x}{\sqrt{5 - 4x - x^2}} \, dx$

3. (7 points) Evaluate $\displaystyle\int \frac{x^4+1}{x^3+2x^2}\,dx,$ showing all reasoning.

4. (12 points)

(a) Evaluate $\int_0^1 \frac{z}{\sqrt{1-z^2}}\,dz$ or explain why its value does not exist; show all reasoning.

(b) Determine whether $\int_{-\infty}^{\infty} \frac{\arctan x}{\sqrt{1+x^2}}\,dx$ converges or diverges; give complete reasoning.

5. (6 points) The linear density of a rod of length 6 cm is given by $d(x)$, in grams per centimeter, where x is measured in centimeters from one end of the rod. Here is a table of values of $d(x)$ measured at one-centimeter intervals:

x (cm)	0	1	2	3	4	5	6
$d(x)$ (g/cm)	1	3	7	11	14	16	17

(a) Use the Trapezoidal Rule to estimate the total mass of the rod. Use all the data in the table above, and do not simplify your answer.

(b) Use Simpson's Rule to estimate the total mass of the rod. Use all the data in the table above, and do not simplify your answer.

6. (12 points) Let $f(x) = \dfrac{6}{\sqrt{1-x^2}}$. In this problem, we study approximations of π using the identity:

$$\pi = \int_0^{\frac{1}{2}} \frac{6}{\sqrt{1-x^2}}\,dx$$

and the Midpoint Rule. (By the way, you do *not* have to prove the above identity.)

(a) Write an algebraic expression involving only numbers that approximates π using the Midpoint Rule with 5 subintervals. You do *not* have to simplify this expression.

(b) Show that the above approximation is accurate to within $\frac{1}{200}$; and explain whether the approximation gives an overestimate or underestimate of π (or whether it is impossible to tell). You may make use of the fact that $f''(x) = \dfrac{6(1+2x^2)}{(1-x^2)^{5/2}}$.

(c) Again using the Midpoint Rule, how many subintervals n would guarantee an approximation of π that is accurate to within 10^{-12}? Your final answer should give a valid n in simplified form, and be fully justified, but it need not be optimal in any sense.

7. (10 points) Let R be the region in the xy-plane lying below the curve $y = \sqrt{x}\cos x$ and above that portion of the x-axis with $0 \leq x \leq \frac{\pi}{2}$.

   (a) Set up, but do not yet evaluate, an integral in terms of a single variable that represents the volume of the solid obtained by rotating R about the x-axis. Justify your answer by drawing a picture, labeling a sample slice, and citing the method used.

   (b) Evaluate the integral of part (a), showing all your steps.

8. (10 points) Consider the region R in the xy-plane bounded by the curves

$$x = 1 - y^2 \qquad \text{and} \qquad x = y^4 - 1$$

(a) Set up, but do not evaluate, an integral in terms of a single variable that represents the area of R. Justify your answer by drawing a picture and labeling a sample slice.

(b) Set up, but do not evaluate, an integral in terms of a single variable that represents the volume of the solid obtained by rotating R around the line $y = 1$. Justify your answer by citing the method used, drawing a picture and labeling a sample slice.

9. (6 points) A spherical planet has radius R kilometers. The planet's atmosphere, also spherical in shape, extends from its surface up to an altitude of A kilometers above the surface. The density of the atmosphere at an altitude h kilometers above the surface of the planet can be given approximately by

$$\rho(h) = A - h \quad (\text{kg/km}^3)$$

Set up, but do not evaluate, an integral in terms of a single variable that represents the total mass of the planet's atmosphere (in kg). Justify your answer by describing how to slice up the atmosphere into thin pieces, each of approximately uniform density.

Remark. Some formulas that may or may not be useful: the volume V and surface area S of a sphere of radius r are given, respectively, by

$$V = \frac{4}{3}\pi r^3, \qquad S = 4\pi r^2.$$