Math 42: Calculus
Second Exam — February 23, 2010

Name: ____ **SUID#:** ______________

Circle your section:				
Sukhada Fadnavis 02 (10-10:50am) 07 (1:15-2:05pm)	Dan Jerison 03 (11-11:50am) 08 (1:15-2:05pm)	Jason Lo 04 (11-11:50am) 09 (1:15-2:05pm)	Dmitriy Ivanov 05 (11-11:50am) 10 (2:15-3:05pm)	Yu-jong Tzeng ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 13 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, March 9**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	10		**6**	12	
2	9		**7**	9	
3	10		**8**	9	
4	9		**9**	12	
5	20		**Total**	100	

1. (10 points) A certain Internet search company owns a data center consisting of thousands of computer hard drives, any of which could fail at any time. The engineers have determined that for the drives they use, the probability density function for the lifespan of a random hard drive is given by

$$f(t) = \begin{cases} Ct & \text{if } 0 \le t \le 2, \\ 2Ce^{-(t-2)/3} & \text{if } t > 2, \\ 0 & \text{otherwise,} \end{cases}$$

where t is measured in years, and C is a positive constant.

(a) Find C, using the fact that f is a probability density function.

(b) Find the mean lifespan of a hard drive used by the company.

2. (9 points) Determine with justification whether each series converges, and if so, find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n/2}}{2^{n+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{8^n} + \ln\left(\frac{2n+3}{2n+1} \right) \right)$

3. (10 points) Suppose that the series $\sum_{n=1}^{\infty} a_n$ converges, for *positive* numbers a_n.

Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ Converges Diverges Inconclusive

(b) $\sum_{n=1}^{\infty} \frac{e^{a_n}-1}{a_n}$ Converges Diverges Inconclusive

(c) $\sum_{n=1}^{\infty} \sin(a_n)$ Converges Diverges Inconclusive

(d) $\sum_{n=1}^{\infty} \sqrt{n}\, a_n$ Converges Diverges Inconclusive

(e) $\sum_{n=1}^{\infty} (n\, a_n - 1)$ Converges Diverges Inconclusive

4. (9 points) For this problem, we consider the series $s = \sum_{n=1}^{\infty} \frac{1}{(n+1) \cdot (\ln(n+1))^2}$.

(a) Explain why this is a convergent series; that is, explain why the number s is defined.

(b) Let s_{100} stand for the sum of the first 100 terms of the series. Determine the accuracy of using s_{100} as an approximation for s. State your conclusion in a complete sentence, and be as quantitatively precise as you can.

(c) It turns out that $s_{100} = 1.8933...$ Based on your reasoning from part (b), find a more accurate approximation for s, *without* having to consider any more terms from the series, and precisely state the accuracy of your new approximation. Your answers do not need to be simplified.

5. (20 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n + 3n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(-2)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^3}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$

6. (12 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(5x-3)^n}{n\,5^n}$$

7. (9 points) Let $f(x) = x^{3/2}$.

(a) Find the degree-2 Taylor polynomial T_2 for f about 9.

(b) Use T_2 to find an approximation for $(9.1)^{3/2}$.

(c) Determine the accuracy of your approximation from part (b), explaining the steps of your reasoning, and giving your final conclusion in sentence form.

8. (9 points)

(a) Find, showing all your steps, the Taylor series for $\sin x$ with center 0.

(b) Use series to find $\lim_{x\to 0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right)$. (You may take for granted the fact that the Taylor series for $\sin x$ converges to $\sin x$.)

9. (12 points) In each of the parts below, show all the steps in your reasoning.

   (a) Write $\dfrac{1}{1+x}$ as a power series about 0, and state the interval of convergence. (Hint: use geometric series.)

   (b) Find a power series for $\ln(1+x)$. What is the radius of convergence?

(c) Express the number $\int_0^{0.01} \frac{\ln(1+x)}{x}\,dx$ as a series.

(d) How many terms of the series of part (c) are required to estimate the integral to within 0.000005, or 5×10^{-6}? Explain completely.