Math 42: Calculus
First Exam — January 28, 2010

Name: ______________________________ **SUID#:** ____________

Select your section:				
Sukhada Fadnavis 02 (10-10:50am) 07 (1:15-2:05pm)	Dan Jerison 03 (11-11:50am) 08 (1:15-2:05pm)	Jason Lo 04 (11-11:50am) 09 (1:15-2:05pm)	Dmitriy Ivanov 05 (11-11:50am) 10 (2:15-3:05pm)	Yu-jong Tzeng ACE

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 11 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, February 11**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ____________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	40		**5**	8	
2	8		**6**	10	
3	10		**7**	8	
4	8		**8**	8	
			Total	100	

1. (40 points) Evaluate each of the following integrals, showing all of your reasoning.

   (a) $\displaystyle\int_3^8 x\sqrt{1+x}\,dx$

   (b) $\displaystyle\int e^t \cos t\,dt$

(c) $\int x^2\sqrt{9-x^2}\,dx$

(d) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

(e) $\displaystyle\int \frac{1}{(t+2\sqrt{t}+2)\sqrt{t}}\,dt$

(f) $\displaystyle\int \frac{2x^2}{x^3 - x^2 - x + 1}\,dx = \int \frac{2x^2}{(x-1)^2(x+1)}\,dx$

2. (8 points)

(a) Set up an integral that represents the length of the curve $y = e^{2x} + 3$ from the point $(0, 4)$ to the point $(1, 3 + e^2)$. Show your steps, but stop before evaluating the integral.

(b) Now evaluate the integral you found in part (a); you do not have to simplify the numerical expression you obtain. You may find it useful to know the following integral table entry, which you *do not* have to prove:

$$\int \frac{\sqrt{a^2 + u^2}}{u}\, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

3. (10 points) Consider the integral $\int_0^1 f(x)\,dx$, where $f(x) = \sqrt{1+x^3}$.

(a) Estimate the error made in approximating the value of this integral using the Trapezoidal Rule with $n = 10$ subintervals. *State your answer in a complete sentence.* You may make use of the fact that $f''(x) = \dfrac{3x(x^3+4)}{4(x^3+1)^{3/2}}$.

(b) Would the Trapezoidal Rule approximation described in part (a) give an overestimate or underestimate of the actual value, or is it impossible to tell with the information given? Explain briefly.

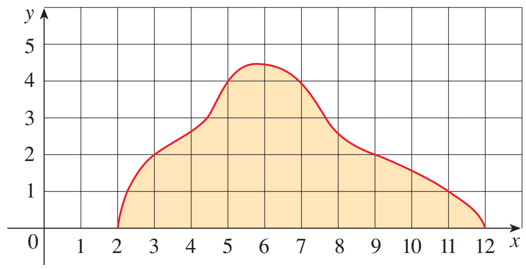
(c) Again using the Trapezoidal Rule, how many subintervals n would be necessary to guarantee an error of at most 10^{-6}? Give a valid n in simplified form. (As long as you justify your answer, you do not have to worry about finding the best possible value.)

4. (8 points)

(a) Set up, but do not evaluate, an integral representing the area of the region bounded by the curves $y = 5 - x^2$ and $y = x^2 + 3x + 3$. As justification, draw a picture with a sample slice labeled.

(b) Set up an integral representing the volume obtained by rotating the region from part (a) about the x-axis. Make sure you justify your answer (draw and label a diagram). Again, don't evaluate the integral.

5. (8 points) Consider the region between the curve and the x-axis in the figure below:



(a) If the region shown is rotated about the x-axis to form a solid, use Simpson's Rule with $n = 10$ to estimate the volume of the solid. (Write an expression involving only numbers, but you do not have to evaluate the expression.)

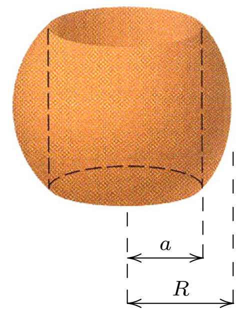
(b) If the region is instead rotated about the y-axis, use the Midpoint Rule with $n = 5$ to estimate the volume of the resulting solid. (Again, you do not have to evaluate your expression.)

6. (10 points)

(a) Set up two distinct integrals, each in terms of a single variable, representing the area of the region in the first quadrant bounded by the curves $y = x^3$ and $y = \sqrt{x}$. For each, justify your answer by drawing a picture and marking a sample slice. Don't evaluate either integral.

(b) Set up two distinct integrals, each in terms of a single variable, representing the volume of the solid obtained by rotating the region from part (a) around the line $x = 1$. For each, make sure you justify your answer (draw a picture, label a sample slice, and cite the method used). Don't evaluate either integral.

7. (8 points) A napkin ring is made by taking a solid wooden ball (sphere) of radius R and drilling a hole of radius a straight through the center. (The hole is cylindrical in shape with radius a, and the resulting solid has flat edges at its top and bottom.) Find the volume of the napkin ring, in terms of R and a.



8. (8 points) A fuel tank buried underground has the shape of a circular cylinder lying lengthwise (so that the axis of symmetry is *horizontal*, i.e., parallel to the ground). The radius of the cylinder is 6 feet, the length is 30 feet, and the top of the tank is 4 feet below ground level.

   Suppose that the tank is completely filled with a liquid that has weight density 40 lb/ft^3. Set up an integral in terms of a single variable that represents the amount of work it takes to pump *half* of the liquid out of the tank and up to ground level. Show all your steps in setting up the integral, but do not evaluate it.