Math 42: Calculus
Second Exam — February 24, 2009

Name : ____

Section Leader: (Circle one)	Fai Chandee	Sukhada Fadnavis	Ha Pham	Ian Weiner	Ziyu Zhang
Section Time: (Circle one)	10:00	11:00	1:15	2:15	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 14 pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Tuesday, March 10**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ________________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	10		**6**	12	
2	10		**7**	10	
3	8		**8**	8	
4	20		**9**	12	
5	10		**Total**	100	

1. (10 points) Determine whether each of the following improper integrals converges. Explain your reasoning completely.

   (a) $\displaystyle\int_{-\infty}^{\infty} \sin(2x)\,dx$

   (b) $\displaystyle\int_{0}^{1} \frac{e^x}{\sqrt{x}}\,dx$

2. (10 points) For this problem, use the following information about any normal ("bell-shaped" or "Gaussian") probability density function f:

- f has the general form $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
- $\displaystyle\int_{-\infty}^{\mu+\sigma} f(x)dx \approx .84$
- $\displaystyle\int_{-\infty}^{\mu+2\sigma} f(x)dx \approx .98$

Now suppose that the speeds of vehicles on a highway with speed limit 65 mph are normally distributed, with mean 70 mph and standard deviation 5 mph.

(a) What is the probability that a randomly chosen vehicle is traveling at a legal speed (65 or under)? Justify your answer by writing an integral expression that represents this probability and showing how to evaluate this integral.

(b) If the Highway Patrol are instructed to ticket motorists driving 80 mph or more, what percentage of motorists are targeted? Again use an integral to express your answer, and evaluate it with justification.

3. (8 points)

(a) Express $1.53\overline{42} = 1.53424242\ldots$ as a ratio of integers.

(b) Determine the values of b for which $1 + \frac{e^{-2b}}{2} + \frac{e^{-4b}}{4} + \frac{e^{-6b}}{8} + \frac{e^{-8b}}{16} + \cdots$ converges, and for those b find the sum. (Your answer will be an expression in terms of b.)

4. (20 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3n^4 - n^2 + 5}$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{1}{3^n}\right)^2$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$

5. (10 points) Suppose that the series $\sum_{n=1}^{\infty} a_n$ converges, for *positive* numbers a_n.

   Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

   (a) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ Converges Diverges Inconclusive

   (b) $\sum_{n=1}^{\infty} (a_n)^3$ Converges Diverges Inconclusive

   (c) $\sum_{n=1}^{\infty} e^{a_n}$ Converges Diverges Inconclusive

   (d) $\sum_{n=1}^{\infty} (-1)^n a_n$ Converges Diverges Inconclusive

   (e) $\sum_{n=1}^{\infty} \sqrt{a_n}$ Converges Diverges Inconclusive

6. (12 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$$

7. (10 points) Match each function below with its power series, listed among the choices below. You do not need to justify your answers. (Not all of the series listed have a match.)

(a) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(b) $\frac{1}{10} + \frac{x}{100} + \frac{x^2}{1000} + \frac{x^3}{10000} + \cdots$

(c) $1 - x^2 + x^4 - x^6 + \cdots$

(d) $1 - \frac{2^2}{2!}x^2 + \frac{2^4}{4!}x^4 - \frac{2^6}{6!}x^6 + \cdots$

(e) $2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \cdots$

(f) $\frac{x}{9} + \frac{x^3}{9^2} + \frac{x^5}{9^3} + \frac{x^7}{9^4} + \cdots$

(g) $x^2 - 3x^3 + \frac{3^2}{2!}x^4 - \frac{3^3}{3!}x^5 + \cdots$

(h) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots$

(i) $\frac{1}{3}x^3 - \frac{3}{4}x^4 + \frac{3^2}{5 \cdot 2!}x^5 - \frac{3^3}{6 \cdot 3!}x^6 + \cdots$

(j) $\frac{1}{10} - \frac{x}{100} + \frac{x^2}{1000} - \frac{x^3}{10000} + \cdots$

Function	Series (choose one of (a) through (j))
$\frac{x}{9 - x^2}$	
$\frac{1}{10 - x}$	
$x^2 e^{-3x}$	
$\frac{1}{(1+x)^2}$	
$\int_0^x t^2 e^{-3t}\,dt$	

8. (8 points) Suppose that f is a function with continuous derivatives and $f(5) = 3$, $f'(5) = -2$, $f''(5) = 1$, and $f'''(5) = -3$.

   (a) Determine the degree-3 Taylor polynomial T_3 of f about 5.

   (b) Use the Taylor polynomial that you found in part (a) to approximate $f(4.9)$. Express your answer as a number (or sum of numbers).

   (c) Suppose $|f^{(4)}(x)| \leq 2$ on the interval $[4.9, 5.1]$. Use this information to make a statement about the accuracy of the approximation that you found in part (b).

9. (12 points)

(a) Compute a power series expansion for $\cos x$, centered at 0. (Show all of your steps.)

(b) Find a power series expansion for $\displaystyle\int \frac{1-\cos x}{x^2}\,dx$ and determine its radius of convergence.

(c) Use your answer in part (b) to find a series for $\displaystyle\int_0^1 \frac{1-\cos x}{x^2}\,dx$.

(d) If you approximate the definite integral $\int_0^1 \frac{1-\cos x}{x^2}\,dx$ by taking the partial sum consisting of the first four nonzero terms of the series that you obtained in part (c), what can you say about the accuracy of your approximation? Be as precise as you can, and state your reasoning clearly.