Math 42: Calculus
First Exam — January 29, 2009

Name : _____

Section Leader: (Circle one)	Fai Chandee	Sukhada Fadnavis	Ha Pham	Ian Weiner	Ziyu Zhang
Section Time: (Circle one)	10:00	11:00	1:15	2:15	

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 13 pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **Thursday, February 12**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: _____

The following boxes are strictly for grading purposes. Please do not mark.

1	42		**5**	10	
2	8		**6**	10	
3	6		**7**	8	
4	8		**8**	8	
			Total	100	

1. (42 points) Evaluate each of the following integrals, showing all of your reasoning.

   (a) $\int t^2 \cos 2t \, dt$

   (b) $\int \frac{dx}{x \ln x \ln(\ln x)}$

(c) $\displaystyle\int \frac{z^2+z-1}{z^2+1}\,dz$

(d) $\displaystyle\int_{\sqrt{10}}^{5} \frac{1}{x^2\sqrt{x^2-9}}\,dx$

(e) $\displaystyle\int_0^{\pi/2} \sin^5\theta \, \cos^4\theta \, d\theta$

(f) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}} \, dx$

2. (8 points) Show all steps in evaluating the integral $\displaystyle\int \frac{x^4+1}{x^3-x^2-2x}\,dx = \int \frac{x^4+1}{x(x+1)(x-2)}\,dx.$

3. (6 points) A particle is moved along the x-axis by a force that measures $F(x)$ Newtons at a point x meters from the origin, where values of $F(x)$ are given in the table below.

x	0	2	4	6	8	10	12	14	16
$F(x)$	10.5	9.6	8.8	8.1	7.5	7.0	6.7	6.5	6.4

(a) Using the *Midpoint Rule*, write a sum using *values from the chart* representing an estimate for the work done by the force in moving the object a distance of 16 m. You do not have to simplify your expression.

(b) Do the same using *Simpson's Rule*; again, you do not have to simplify the expression.

4. (8 points) Consider the integral $\int_0^2 f(x)\,dx$, where $f(x) = e^{-x^2/2}$.

   (a) Estimate the error made in approximating the value of this integral using the Trapezoidal Rule with $n = 5$ subintervals. *State your answer in a complete sentence.* You may make use of the fact that $f''(x) = (x^2 - 1)e^{-x^2/2}$.

   (b) Again using the Trapezoidal Rule, how many subintervals n would be necessary to guarantee an error of at most 10^{-4}? Give a valid n in simplified form. (As long as you justify your answer, you do not have to worry about finding the best possible value.)

5. (10 points) Consider the region R bounded by the curves $y = x$, $y = x^2 + 1$, $x = 0$, and $x = 2$.

   (a) Set up, but do not evaluate, an integral that gives the area of the region R. Justify your answer by drawing a picture and marking a sample slice.

(b) Set up an integral representing the volume of the solid obtained by rotating the region R about the x-axis. Justify your answer by drawing and labeling a picture with a sample slice, and cite the method used, but don't evaluate the integral.

(c) If the base of a solid V is the region R, and if all cross-sections of V that are perpendicular to the x-axis are squares, write a definite integral that will give the volume of V. You don't have to evaluate the integral.

6. (10 points)

   (a) Set up, but do not evaluate, an integral representing the area of the region bounded by the curves $x = 6 - y^2$ and $y = -x$. As justification, draw a picture with a sample slice labeled.

(b) Set up an integral representing the volume of the solid obtained by rotating the region from part (a) around the line $x = -3$. Make sure you justify your answer (draw and label a diagram, and cite the method). Again don't evaluate the integral.

(c) Now do the same for the volume of the solid obtained by rotating the region from part (a) around the line $y = 4$.

7. (8 points) A 300-foot cable with a linear density of 1.3 lb/ft is suspended by one end from the top of a 200-foot building — so that the other end lies on the ground, which is completely level.

   Suppose that we wish to lift the cable's lower end up to the top of the building, while keeping the upper end fixed there, so that the two ends of the cable are level with each other (and the remainder of the cable hangs below, now doubled up). How much work is required to accomplish this? Show all the steps in your computation.

8. (8 points) The Great Pyramid of Giza is roughly a solid square pyramid made of limestone; when first constructed it stood 500 feet tall, with a square base of side length 800 feet. (Like any square pyramid, each horizontal cross-section of the pyramid is a square.) The weight density of limestone is 150 lb/ft^3. How much work was done to lift upward, from ground-level, all the limestone that forms this pyramid?