Math 42: Calculus
Second Exam — March 1, 2007

Name : ____

Section Leader (Circle one) : Buyukboduk Chang Lee Segerman Zhang

Section Time (Circle one): 11:00 1:15

- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answer. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	10		**6**	13	
2	10		**7**	8	
3	6		**8**	9	
4	24		**9**	14	
5	6		**Total**	100	

1. (10 points) At the Stanford Federal Credit Union, the waiting time for teller service is a randomly varying quantity that can be modeled by the following probability density function:

$$f(x) = \begin{cases} \frac{1}{3}e^{-x/3} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0, \end{cases}$$

where x is measured in minutes. It is a fact that the mean waiting time is 3 minutes, according to this PDF. (You do not have to prove this fact!)

   (a) Calculate the probability that an experience waiting in line takes longer than average; that is, takes longer than 3 minutes.

   (b) Determine the median waiting time, showing all reasoning.

2. (10 points)

(a) Find $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$.

(b) Determine whether $\displaystyle\int_1^\infty \frac{2}{1+x^5}\,dx$ converges. Explain your reasoning completely.

3. (6 points) A dosage d of a drug is given daily at $t = 0, 1, 2, 3, \ldots$ days. The drug decays exponentially at a rate k in the bloodstream. Thus, the amount in the bloodstream after $n + 1$ doses is $d + de^{-k} + de^{-2k} + \cdots + de^{-nk}$.

   (a) Find the level of the drug after "infinitely many" doses. That is, find

$$d + de^{-k} + de^{-2k} + \cdots + de^{-nk} + \cdots$$

   (b) If $k = 0.1$, what dosage is needed to maintain a drug level of 2?

4. (24 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^4 + n + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^3}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{2^n}{n!\,\ln(n+1)}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

5. (6 points) Suppose we know that $\sum_{n=1}^{\infty} a_n$ converges to 0.8. We are given no other information about this series. For each of the following statements, circle

- **T** if the statement must be true,
- **F** if the statement must be false, and
- **X** if the statement could be either true or false.

You do not need to justify your answers.

T **F** **X** $\lim_{n\to\infty} a_n = 0.8.$

T **F** **X** $\lim_{n\to\infty} a_n = 0.$

T **F** **X** $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1.$

T **F** **X** $a_{n+1} < a_n$ for all n.

T **F** **X** $\lim_{n\to\infty} \frac{1}{|a_n|} = \infty.$

T **F** **X** $\lim_{n\to\infty} s_n = 0.8$, where $s_n = a_1 + a_2 + \cdots + a_n$.

6. (13 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{3^n \sqrt{n}}$$

7. (8 points) Consider the following power series:

(I) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$

(II) $1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \cdots$

(III) $\frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \cdots$

(IV) $x - x^3 + x^5 - \cdots$

(V) $x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} - \cdots$

Fill in the letter of the series that corresponds to the given function. You do not need to justify your answer. (One of the above series does not have a match.)

Function	I, II, III, IV, or V
$1 - \cos x$	
$x^2 \cos x$	
$\sin(x^2)$	
$\frac{x}{1+x^2}$	

8. (9 points) Let f be a function all of whose derivatives (first, second, etc.) are defined everywhere. Suppose the degree-3 Taylor polynomial for f about -2 is given by

$$T_3(x) = 2 - \frac{3}{8}(x+2)^2 - \frac{1}{12}(x+2)^3.$$

(a) Find $f'(-2)$, $f''(-2)$, and $f'''(-2)$.

(b) Use T_3 to find an approximation for $f(0)$.

(c) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \leq \frac{1}{4}$ over the interval $[-4, 0]$. Use this information to make a statement about the accuracy of the approximation for $f(0)$ that you found in part (b).

9. (14 points)

(a) Compute a power series expansion for e^x, centered at 0. (Show all of your steps.)

(b) Find a power series expansion for $\int e^{-x^3}\,dx$ and determine its radius of convergence.

(c) Use your answer in part (b) to find a series for $\displaystyle\int_0^{1/2} e^{-x^3}\,dx$.

(d) If you approximate the definite integral $\int_0^{1/2} e^{-x^3}\,dx$ by taking the partial sum consisting of the first four nonzero terms of the series that you obtained in part (c), what can you say about the accuracy of your approximation? Be as precise as you can, and state your reasoning clearly.