Math 42: Calculus
First Exam — February 1, 2007

Name : ____

Section Leader (Circle one) : Buyukboduk Chang Lee Segerman Zhang

Section Time (Circle one): 11:00 1:15

- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answer. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	40	
2	10	
3	6	
4	8	
5	10	
6	10	
7	8	
8	8	
Total	100	

1. (40 points) Evaluate each of the following integrals, showing all of your reasoning.

(a) $\displaystyle\int_3^4 \frac{x}{\sqrt{25 - x^2}}\, dx$

(b) $\displaystyle\int_1^{e^2} \ln z \, dz$

(c) $\int \cos^3 2t \, dt$

(d) $\int z \arctan z \, dz$

(e) $\int \sin^2 4x \, dx$

(f) $\int (4 - x^2)^{-3/2} \, dx$

2. (10 points) Evaluate the integral $\int \frac{4x^3}{x^3 - x^2 + 3x - 3}\,dx = \int \frac{4x^3}{(x-1)(x^2+3)}\,dx.$

3. (6 points) A rocket has vertical position 0 at time 0. The following chart gives the rocket's upward velocity, in meters per second, at time t seconds.

t	0	2	4	6	8	10	12
v	1	8	25	60	120	200	350

(a) Using all the data in the chart, write a sum representing an estimate of the rocket's height at time $t = 12$, using the *Trapezoidal Rule*. You do not have to simplify or fully evaluate your expression.

(b) Do the same using *Simpson's Rule*; again, you do not have to simplify the expression.

4. (8 points) Consider the integral $\int_0^2 \cos(x^2)\,dx$.

(a) Estimate the error made in approximating the value of this integral using the Midpoint Rule using $n = 6$ subintervals. *State your answer in a complete sentence.*

(b) Again using the Midpoint Rule, how many subintervals n would be necessary to guarantee an error of at most $\frac{1}{1000}$? Give a valid n in simplified form. (As long as you justify your answer, you do not have to worry about finding the best possible value.)

5. (10 points)

(a) Set up two distinct integrals, each in terms of a single variable, representing the area of the region in the first quadrant bounded by the curves $y = x^2$ and $y = x^{1/3}$. For each, make sure you justify your answer (draw a picture and mark a sample slice). Don't evaluate either integral.

(b) Set up two distinct integrals, each in terms of a single variable, representing the volume obtained by rotating the region from part (a) around the x-axis. For each, make sure you justify your answer (draw a picture, label a sample slice, and cite the method used). Don't evaluate either integral.

6. (10 points)

(a) Set up, but do not evaluate, an integral representing the area of the region bounded by the curves $x = y^2 - 2$ and $y = -x$. As justification, draw a picture with a sample slice labeled.

(b) Set up an integral representing the volume obtained by rotating the region from part (a) around the line $x = 2$. Make sure you justify your answer (draw and label a diagram, and cite the method). Again, don't evaluate the integral.

7. (8 points) One very rainy day, a bucket is raised from ground level to the top of a building 200 ft high, using a rope having a linear density of 0.1 lb/ft. Initially (at ground level), the bucket weighs 5 pounds; however, with rain continuing to pour at a constant rate, the bucket takes on water and actually weighs 10 pounds by the moment it reaches the top of the building. Assuming the bucket is being raised at a constant rate, how much work is required to pull the bucket (with its rope) to the top?

8. (8 points) A giant clay ant hill in northeastern Argentina has the shape of a perfect cone, with its circular base on the ground. The base has a 1-foot radius, and the ant hill is 3 feet in height. The density of the material is a uniform 30 lb/ft^3. How much work have the ants done to assemble this ant hill, lifting all material vertically from ground-level?

Formulas for Reference

Trig Formulas:

$$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$$

$$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

Error-Bound Formulas for Approximate Integration:

$$|E_T| \leq \frac{K_2(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K_2(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{K_4(b-a)^5}{180n^4}$$