Math 42 — Practice Problems for Exam 2

Note: Please be sure also to check out the collection of **probability** questions from old exams — linked where you found this practice exam.

1. Evaluate the following integrals, showing all work.

   (a) $\displaystyle\int_0^1 \frac{1}{\sqrt[5]{x}}\,dx$

   (b) $\displaystyle\int_0^2 \frac{1}{\sqrt{x}}\,dx$

2. Determine whether the improper integral $\displaystyle\int_3^\infty \frac{\ln x}{\sqrt{x}}\,dx$ converges or diverges.

3. Determine whether the improper integral $\displaystyle\int_1^\infty \frac{\cos^2 x}{x^3}\,dx$ converges or diverges.

4. [*Deleted for 2007*]

5. True / False: (You do not need to justify your answer.)

   (a) $\displaystyle\int_1^\infty \frac{1}{x^{\sqrt{2}}}\,dx$ is convergent.

   (b) If $\sum c_n 2^n$ is divergent, then $\sum c_n(-3)^n$ is divergent.

   (c) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

6. Find the sums of the following series.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n(n+2)}$

   (b) $\displaystyle\sum_{k=1}^\infty \frac{2^{k-1}-3}{5^{k+1}}$

7. Determine whether each of the following series converges.

   (a) $\displaystyle\sum_{n=1}^\infty e^{-1/n}$

   (b) $\displaystyle\sum_{n=1}^\infty \frac{2n}{(n+3)^{3/2}}$

   (c) $\displaystyle\sum_{n=1}^\infty ne^{-n}$

   (d) $\displaystyle\sum_{k=1}^\infty \frac{3}{k^2+7}$

8. Determine whether each of the following series converges or diverges.

   (a) $\displaystyle\sum_{n=2}^\infty \frac{1}{n\ln n}$

   (b) $\displaystyle\sum_{n=0}^\infty (-1)^n \frac{n^3+3n+2}{n^3+6}$

   (c) $\displaystyle\sum_{n=1}^\infty \frac{n}{2^n(n+1)}$

9. Find the sums of each of the following series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2^n}{4^n}$

10. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$.

11. Find a power series expansion, centered at 0, for $f(x) = \dfrac{x}{2+x}$ and its radius of convergence.

12. Isaac Newton showed that $(1-x^2)^{-1/2} = \displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^{2n}$ for $-1 < x < 1$.

    (a) Using this formula, find a power series expansion for $\arcsin x$.

    (b) Use your power series from part (a) with $x = 1/2$ to find an infinite series whose sum is π.

13. Use power series expansions to compute $\displaystyle\lim_{x \to 0} \frac{e^{x^2} - 1}{\cos x - 1}$.

14. (a) Find the third-degree Taylor polynomial for $f(x) = x^{4/3}$ about $a = 27$.

    (b) Estimate the maximum error involved in estimating f with the Taylor polynomial you found in part (a) for $25 \le x \le 29$.