# Math 42 Practice Exam 1

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| **2007 Notes:** | Skip problems 1(e), 2, 6(b), and 6(e); See other review resources for problems on "Work" (not covered below). |

1. Evaluate each of the following integrals.

   (a) $\displaystyle\int \frac{r}{r^2-4}\,dr$

   (b) $\displaystyle\int \frac{1}{r^2-4}\,dr$

   (c) $\displaystyle\int_1^e x\ln x\,dx$

   (d) $\displaystyle\int \sin^4 x\,dx$

   (e) $\displaystyle\int_0^1 \frac{1}{\sqrt[5]{x}}\,dx$

   (f) $\displaystyle\int_{\pi/4}^{\pi/2} \sin^2\theta\cos\theta\,d\theta$

   (g) $\displaystyle\int_0^1 \frac{1}{(x^2+9)^{3/2}}\,dx$

   (h) $\displaystyle\int \frac{x^3}{x^3-x^2-x+1}\,dx = \int \frac{x^3}{(x+1)(x-1)^2}\,dx$

   (i) $\displaystyle\int_1^4 (at+b)\sqrt{t}\,dt$ ($a$ and $b$ are constants.)

2. Determine whether the improper integral

$$\int_3^\infty \frac{\ln x}{\sqrt{x}}\,dx$$

   converges or diverges.

3. The following chart gives the rate of poplulation growth (i.e., the number of births minus the number of deaths per year) of a certain small town in the given years.

| Year: | 1980 | 1985 | 1990 | 1995 | 2000 |
|---|---|---|---|---|---|
| Rate of growth: | 1 | $-1$ | 0 | 1 | 2 |

   Use Simpson's Rule to estimate how much the population grew from 1980 to 2000.

4. Consider the integral $\displaystyle\int_0^1 \sin(x^2)\,dx$.

   (a) Estimate the error made in approximating the value of this integral using $n=5$ trapezoids.

   (b) How many trapezoids would be necessary to guarantee an error of at most $\frac{1}{200}$?

5. Consider the region $R$ in the plane bounded between the curves $y = x$ and $y = x^2$.

   (a) Find the area of $R$.

   (b) Find the volume of the solid obtained by revolving $R$ about the $x$-axis.

   (c) Find the volume of the solid obtained by revolving $R$ about the line $x = 1$.

6. True / False: (You do not need to justify your answer.)

   (a) Simpson's Rule usually, but not always, gives a more accurate approximation for a definite integral than both the Midpoint Rule and the Trapezoid Rule.

   (b) If $f(x) \leq g(x)$ and $\displaystyle\int_0^\infty g(x)\, dx$ diverges, then $\displaystyle\int_0^\infty f(x)\, dx$ also diverges.

   (c) $\displaystyle\int \sin^2 x\, dx = \frac{1}{3}\sin^3 x + C$.

   (d) It is possible to antidifferentiate every rational function in terms of finitely many familiar functions.

   (e) If $\displaystyle\int_a^\infty f(x)\, dx$ and $\displaystyle\int_a^\infty g(x)\, dx$ are both divergent, then $\displaystyle\int_a^\infty [f(x) + g(x)]\, dx$ is also divergent.