Math 42: Fall 2015
Midterm 1
October 13, 2015

NAME: ____

Time: **180 minutes**

For each problem, you should write down all of your work carefully and legibly to receive full credit. When asked to justify your answer, you should use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason.
- You are bound by the Stanford Honor Code, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ____

Discussion Section: (Please circle)

9:30-10:20 10:30-11:20 11:30-12:20 12:30-1:20 ACE 1:30-3:20

There are four pages of blank paper at the end of the exam. Please use them for any scratch work and include them with your exam when you turn it in.

Problem	**Value**	**Points**
1	12	
2	10	
3	30	
4	10	
5	12	
6	10	
7	6	
8	10	
Total	100	

1. (Short answer) You **do not** have to justify your answer to the following questions.

a. *(2 pts.)* For what values of p does the *sequence* $\{1/n^p\}$ converge? ____________

b. *(2 pts.)* For what values of p does the *series* $\displaystyle\sum_{n=1}^{\infty} 1/n^p$ converge? ____________

c. *(2 pts.)* (True or false) If $\displaystyle\int_1^\infty f(x)\,dx$ and $\displaystyle\int_1^\infty g(x)\,dx$ both converge, then $\displaystyle\int_1^\infty f(x) + g(x)\,dx$ must also converge.

d. *(2 pts.)* (True or false) If $\displaystyle\int_1^\infty f(x)\,dx$ and $\displaystyle\int_1^\infty g(x)\,dx$ both diverge, then $\displaystyle\int_1^\infty f(x) + g(x)\,dx$ must also diverge.

e. *(4 pts.)* Write out in terms of limits how to decompose the improper integral $\displaystyle\int_0^\infty \frac{\sqrt{x}}{(x-2)^2}\,dx$. **Do not** evaluate the integral.
(To clarify: If I asked this question about $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$, I'd be looking for $\displaystyle\lim_{t\to\infty}\int_1^t \frac{1}{x^2}\,dx$. Your answer should be a sum of things like this.)

2. Evaluate the following indefinite integrals.

a. *(5 pts.)* $\int \ln \sqrt{x}\, dx$

b. *(5 pts.)* $\int t^3 e^{-t^2}\, dt$

3. Determine whether each of the following improper integrals is convergent or divergent. If it's convergent, to what does it converge?

a. *(5 pts.)* $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$

b. *(5 pts.)* $\displaystyle\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$

c. *(5 pts.)* $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$

d. *(5 pts.)* $\displaystyle\int_{0}^{\infty} \frac{1+e^{-x}}{\sqrt{x}}\,dx$

e. *(5 pts.)* $\int_0^\infty x^2 e^{-x}\, dx$

f. *(5 pts.)* $\int_0^1 \ln x\, dx$

4. Suppose that $f(x)$ is a positive, continuous, and decreasing function on $[1, \infty)$ such that $\lim_{x \to \infty} f(x) = 3$. Define

$$a_n = (-1)^n f(n), \quad b_n = \frac{f(n)}{n}, \quad \text{and} \quad c_n = \frac{(-1)^n f(n)}{n}.$$

a. *(5 pts.)* Which of the sequences $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ converge?

b. *(5 pts.)* Which of the series $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$, and $\sum_{n=1}^{\infty} c_n$ converge?

5. For each of the following series, indicate whether it converges or diverges and what test you used.

a. *(3 pts.)* $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{n+4}}{4^{\pi+n}}$

Convergent Divergent Test: ________________

b. *(3 pts.)* $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^3}$

Convergent Divergent Test: ________________

c. *(3 pts.)* $\displaystyle\sum_{n=1}^{\infty} (2+\sin n)^{1/n}$

Convergent Divergent Test: ________________

d. *(3 pts.)* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+\sqrt{n}+1}$

Convergent Divergent Test: ________________

6. a. *(5 pts.)* For which values of p is the series

$$\sum_{n=1}^{\infty} \frac{4n}{(n^p + 1)^3}$$

convergent? Justify your answer fully.

b. *(5 pts.)* For $p = 2$, how many terms are needed to compute the series in part a. to within $1/100$? That is, find N so that $|S - S_N| \leq 1/100$, where

$$S = \sum_{n=1}^{\infty} \frac{4n}{(n^2 + 1)^3} \quad \text{and} \quad S_N = \sum_{n=1}^{N} \frac{4n}{(n^2 + 1)^3}.$$

7. *(6 pts.)* Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$$

converges or diverges, showing all necessary work.

8. Consider the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^{n+2}\sqrt{n+1}}.$$

a. *(5 pts.)* What is its radius of convergence?
b. *(5 pts.)* What is its interval of convergence?