Solutions:Homework 6

1 Chapter 6.2

Problem 12 We use the washer method. The region is between $y = \sqrt{x}$ to the left and $y = x$ to the right. These intersect at $y = 0$ and $y = 1$. Since the axis of rotation $x = 2$ is to the right of the region, the inner radius is $2 - y$, and the outer radius is $2 - y^2$.

The volume is

$$\begin{aligned} V &= \int_0^1 \pi \left[(2 - y^2)^2 - (2 - y)^2 \right] dy \\ &= \pi \int_0^1 (y^4 - 5y^2 + 4y) dy \\ &= \pi \left[\frac{1}{5} y^5 - \frac{5}{3} y^3 + 2y^2 \right]_0^1 \\ &= \frac{8}{15} \pi. \end{aligned}$$

Problem 15 We use the washer method. The curves $x - y = 1$ and $y = x^2 - 4x + 3$ intersect when $x = 1$ or $x = 4$. As the axis of rotation $y = 3$ lies above the region, the inner radius is $3 - (x - 1)$ and the outer radius is

$3-(x^2-4x+3)$. The volume is

$$\begin{aligned} V &= \int_1^4 \pi \left[(3-(x^2-4x+3))^2 - (3-(x-1))^2\right] dx \\ &= \pi \int_1^4 (x^4-8x^3+15x^2+8x-16)dx \\ &= \pi \left[\frac{1}{5}x^5 - 2x^4 + 5x^3 + 4x^2 - 16x\right]_1^4 \\ &= \frac{108}{5}\pi \end{aligned}$$

2 Chapter 6.3

Problem 5 We use cylindrical shells. The shell has height $h(x) = e^{-x^2}$, and the region lies between $x=0$ and $x=1$. Therefore

$$V = \int_0^1 2\pi x \cdot h(x)dx = \int_0^1 2\pi x \cdot e^{-x^2} dx.$$

Doing a substitution $u = x^2, du = 2x \cdot dx$, we have

$$V = \int_0^1 \pi e^{-u} du = \pi \left[-e^{-u}\right]_0^1 = \pi(-e^{-1}+1).$$

Problem 33 We use cylindrical shells. The curves $y=5$ and $y = x + \frac{4}{x}$ intersect at $x=1$ and $x=4$. The height of a shell is $5-(x+\frac{4}{x})$. Note that the axis of rotation is $x=-1$, so the circumference of a shell is $2\pi(x+1)$. Therefore

$$\begin{aligned} V &= \int_1^4 2\pi(x+1)(5-(x+\frac{4}{x}))dx \\ &= 2\pi \int_1^4 (-x^2+4x+1-\frac{4}{x})dx \\ &= 2\pi \left[-\frac{1}{3}x^3 + 2x^2 + x - 4\ln x\right]_1^4 \\ &= 2\pi(12 - 4\ln 4) = 8\pi(3-\ln 4) \end{aligned}$$

3 Chapter 6.4

Problem 9 $x = y^{3/2}$, so $1 + (dx/dy)^2 = 1 + (\frac{3}{2}y^{1/2})^2 = 1 + \frac{9}{4}y$.

$$L = \int_0^1 \sqrt{1 + \frac{9}{4}y} dy = \int_1^{13/4} \frac{4}{9}\sqrt{u} du,$$

by substituting $u = 1 + \frac{9}{4}y, du = \frac{9}{4}dy$. So

$$L = \frac{4}{9} \cdot \frac{2}{3} \left[u^{3/2}\right]_1^{13/4} = \frac{13\sqrt{13} - 8}{27}.$$

Problem 11 $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $y' = \frac{1}{2}x - \frac{1}{2x}$, $1 + (y')^2 = 1 + (\frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}) = \left(\frac{1}{2}x + \frac{1}{2x}\right)^2$. So

$$\begin{aligned} L &= \int_1^2 \frac{1}{2}x + \frac{1}{2x} dx \\ &= \left[\frac{1}{4}x^2 + \frac{1}{2}\ln x\right]_1^2 \\ &= (1 + \frac{1}{2}\ln 2) - (\frac{1}{4} + 0) \\ &= \frac{3}{4} + \frac{1}{2}\ln 2 \\ &= 2\pi\left[-\frac{1}{3}x^3 + 2x^2 + x - 4\ln x\right]_1^4 \\ &= 2\pi(12 - 4\ln 4) = 8\pi(3 - \ln 4) \end{aligned}$$

Problem 25 $y = \ln\cos x, y' = -\tan x, 1 + (y')^2 = 1 + \tan^2 x = \sec^2 x$. So

$$\begin{aligned} L &= \int_0^{\pi/4} \sec x dx \\ &= [\ln|\sec x + \tan x|]_0^{\pi/4} \\ &= \ln\sqrt{2} + 1 - \ln 1 + 0 = \ln\sqrt{2} + 1 \end{aligned}$$