Solutions:Homework 1

1 Chapter 5.7

Problem 15

To compute the integral $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}dt$ consider the substitution $t = sec\theta$ so $dt = sec\theta \tan\theta d\theta$ and therefore

$$\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}dt = \int_0^{\pi/4} \frac{sec\theta \tan\theta}{sec^3\theta \tan\theta}d\theta = \int_0^{\pi/4} cos^2\theta = \frac{1}{2}\int_0^{\pi/4}(1+cos2\theta)d\theta =$$

$$\frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$$

Problem 15 Let $x = 4\sin\theta$ where $-\pi/2 \leq \theta \leq \pi/2$ then

$$dx = 4\cos\theta d\theta$$

and

$$\sqrt{16-x^2} = 4|cos\theta| = 4\cos\theta$$

if $-\pi/2 \leq \theta \leq \pi/2$. Then

$$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}dx = \int_0^{\pi/3} \frac{4^3\sin^3\theta}{4\cos\theta}4\cos\theta d\theta = \int_0^{\pi/3} 4^3\sin^3\theta d\theta =$$

$$4^3\int_0^{\pi/3} \sin\theta(1-cos^2\theta)d\theta = 4^3[-\cos\theta + \frac{1}{3}\cos^3\theta]_0^{\pi/3} = \frac{40}{3}$$

Problem 22 Write

$$\frac{x-4}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}.$$

Multiply both sides by $x^2 - 5x + 6$ to get

$$x - 4 = A(x-3) + B(x-2)$$

and so $A = 2$ and $B = -1$. Thus

$$\int_0^1 \frac{x-4}{x^2-5x+6}dx = \int_0^1 \left(\frac{2}{x-2} - \frac{1}{x-3}\right) dx = [2\ln|x-2| - \ln|x-3|]_0^1 = -3\ln 2 + \ln 3$$

Problem 23 Write

$$\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}.$$

Multiply both sides by $x^2 - 1$ to get

$$1 = A(x-1) + B(x+1)$$

and so $A = -\frac{1}{2}$ and $B = \frac{1}{2}$. Thus,

$$\int_2^3 \frac{1}{x^2-1}dx = \int_2^3 \left(\frac{-1/2}{x+1} + \frac{1/2}{x-1}\right) dx = [-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1|]_2^3 = \frac{1}{2}(\ln 3 - \ln 2)$$

2 Chapter 5.9

Problem 18b $f(x) = e^{1/x}$ and $\Delta x = \frac{2-1}{10} = \frac{1}{10}$ then

$$f'(x) = -\frac{1}{x^2}e^{1/x}, f"(x) = \frac{2x+1}{x^4}e^{1/x}, f^{(3)}(x) = \frac{-4x^3 - 6x^4 - x^6}{x^8}e^{1/x}$$

Notice that $f^{(3)}(x) \leq 0$ if $x \in [1, 2]$ therefore $f"$ is decreasing in that interval, giving $K = 2e$. Hence,

$$|E_T| \leq \frac{3e(2-1)^3}{12(10^2)} = \frac{e}{400} \approx 0.006796$$

and

$$|E_M| \leq \frac{e}{800} \approx 0.003398$$

Problem 18c Take $K = 3e$ as in part b. Then

$$|E_T| \leq \frac{3e}{12n^2} \leq 0.0001 \Leftrightarrow n \geq 83$$

For M_n:

$$|E_M| \leq \frac{3e}{24n^2} \leq 0.0001 \Leftrightarrow n \geq 59$$

Problem 20 From Example 7(b), take $K = 76e$ and get $|E_S| \leq \frac{76e1^5}{180n^4} \leq 0.00001 \Rightarrow n \geq 18.4$. Hence take $n = 20$ (n has to be even). **Problem 36** Consider $f(x) = |x-1|$. Then the are $\int_0^2 f(x)dx = 1$ and $R_2 = 1$ as well. But, Simpsons rule gives $S_2 = \frac{2}{3}$.

3 Chapter 6.1

Problem 7

$$A = \int_0^1 (\sqrt{x} - x^2)dx = \frac{1}{3}$$

Problem 8 The curves intersect at $x^2 - 2x = x + 4 \Leftrightarrow x^2 - 3x - 4 = 0 \Leftrightarrow x = -1, 4$. Therefore

$$A = \int_{-1}^4 [x + 4 - (x^2 - 2x)]dx = \frac{125}{6}$$

Problem 18 The curves $y = 3x^2$ and $y = -4x + 4$ intersect at $3x^2 = -4x + 4 \Leftrightarrow (3x-2)(x+2) = 0 \Rightarrow x = 2/3$ since $x \geq 0$. The curves $y = 8x^2$ and $y = -4x+4$ intersect at $8x^2 = -4x+4 \Leftrightarrow (2x-1)(x+1) = 0 \Rightarrow x = 1/2$ since $x \geq 0$. Therefore

$$A = \int_0^{1/2} (8x^2 - 3x^2)dx + \int_{1/2}^{2/3} (-4x + 4 - 3x^2)dx = \frac{17}{54}$$

Problem 27 If x = distance from left end of the pool and $w(x)$ is the width at x then we use Simpsons Rule (with $n = 8$ and $\Delta x = 2$) to approximate the area

$$\int_0^{16} w(x)dx \approx$$

$$\frac{2}{3}(0 + 4(6.2) + 2(2.7) + 4(6.8) + 2(5.6) + 4(5.0) + 2(4.8) + 4(4.8) + 0)$$

$$\approx 84m^2$$

Problem 45 The curves $y = mx$ and $y = \frac{x}{x^2+1}$ intersect at $mx = \frac{x}{x^2+1} \Leftrightarrow x = 0$ or $mx^2 + m - 1 = 0$. So $x = 0$ or $x^2 = \frac{1-m}{m}$. Therefore $0 < m < 1$ and

the intersection points are for $x = 0, \pm\sqrt{\frac{m-1}{m}}$. Graphing the region, we see that we get two symmetric sub-regions (the one for negative x and the one for the positive ones) and so the area is

$$A = 2\int_0^{\sqrt{\frac{m-1}{m}}} \left(\frac{x}{x^2+1} - mx\right) dx = [\ln(x^2+1) - \frac{x^2}{2}]_0^{\sqrt{\frac{m-1}{m}}} = m + \ln(1/m) - 1$$

4 Chapter 6.2

Problem 31

$$V = \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx = \frac{\pi r^2 h}{3}$$

Problem 33 If at height y we have radius x notice that Pythagoras' Theorem says that $x^2 + y^2 = r^2 \Rightarrow x^2 = r^2 - y^2$. Therefore,

$$V = \int_{r-h}^{r} (r^2 - y^2) dy = \frac{1}{3}\pi h^2 (3r - h)$$