Solutions:Homework 1

1 Chapter 5.2

Problem 34

The integral $\int_{-2}^{2}\sqrt{4-x^2}dx$ can be viewed as the area below the positive function $f(x)=\sqrt{4-x^2}$. The graph of f from -2 to 2 is just a semicircle, centered at 0 with radius 2. Therefore, $\int_{-2}^{2}\sqrt{4-x^2}dx = \pi 2^2/2 = 2\pi$.

2 Chapter 5.3

Problem 22

$\int_0^1 \frac{4}{t^2+1}dt = 4[\arctan t]_0^1 = 4\arctan(1) - 4\arctan(0) = 4\frac{\pi}{4} - 0 = \pi$.

3 Chapter 5.5

Problem 19

For the integral $\int e^x\sqrt{1+e^x}dx$ consider the u-substitution $u = 1+e^x$. Then $du = e^x dx$ and therefore $\int e^x\sqrt{1+e^x}dx = \int \sqrt{u}du = \int u^{1/2}du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(e^x+1)^{3/2} + C$

Problem 36

For the integral $\int \frac{x}{1+x^4}dx$ consider the u-substitution $u = x^2$. Then $du = 2xdx$ and therefore $\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\arctan(u) + c = \frac{1}{2}\arctan(x^2) + c$

4 Chapter 5.6

Problem 5

The method used in the following integral is integration by parts: $\int re^{r/2}dr = \int r(2e^{r/2})'dr = 2re^{r/2} - 2\int e^{r/2}dr = 2re^{r/2} - 4e^{r/2} + C$

Problem 9

The method used in the following integral is integration by parts: $\int \ln \sqrt[3]{x}dx = \int \ln x^{1/3}dx = \frac{1}{3}\int \ln xdx = \frac{1}{3}\int (x)' \ln xdx = \frac{1}{3}x\ln x - \frac{1}{3}\int x\frac{1}{x}dx = \frac{1}{3}x\ln x - \frac{1}{3}x + C$

Problem 16

The method used in the following integral is integration by parts (twice applied): $\int_0^1 (x^2+1)e^{-x}dx = \int_0^1 (x^2+1)(-e^{-x})'dx = [-(x^2+1)e^{-x}]_0^1 + 2\int_0^1 xe^{-x}dx = (1-\frac{2}{e}) + 2\int_0^1 x(-e^{-x})'dx = (1-\frac{2}{e}) - 2[xe^{-x}]_0^1 + 2\int_0^1 e^{-x}dx = (1-\frac{2}{e}) - 2\frac{1}{e} + 2[-e^{-x}]_0^1 = 3 - \frac{6}{e}$

Problem 29 For the integral $\int x\ln(x+1)dx$ first consider the u-substitution $u = x+1$. Then $du = dx$ and therefore $\int x\ln(x+1)dx = \int (u-1)\ln udu = \int u\ln udu - \int \ln udu$.

Then $\int \ln udu = \int (x)' \ln udu = u\ln u - \int u\frac{1}{u}du = u\ln u - u + C$ and $\int u\ln udu = \int (u^2/2)'\ln udu = \frac{u^2}{2}\ln u - \frac{1}{2}\int u^2\frac{1}{u}du = \frac{u^2}{2}\ln u - \frac{u^2}{4} + c.$

In total, $\int x\ln(x+1)dx = \frac{u^2}{2}\ln u - \frac{u^2}{4} - u\ln u + u + C = \frac{(x+1)^2}{2}\ln(x+1) - \frac{(x+1)^2}{4} - (x+1)\ln(x+1) + x + 1 + C$

5 Chapter 5.10

Problem 6

$$\int_0^\infty \frac{1}{\sqrt[4]{1+x}}dx = \lim_{t\to\infty}\int_0^t \frac{1}{\sqrt[4]{1+x}}dx$$

and

$$\int_0^t \frac{1}{\sqrt[4]{1+x}}dx = \int_0^t (1+x)^{-1/4}dx = [\frac{4}{3}(1+x)^{3/4}]_0^t = \frac{4}{3}(1+t)^{3/4} - \frac{4}{3}$$

and therefore

$$\int_0^\infty \frac{1}{\sqrt[4]{1+x}}dx = \lim_{t\to\infty}\int_0^t \frac{1}{\sqrt[4]{1+x}}dx = \infty$$

which means that the integral diverges.

Problem 9

$$\int_4^{\infty} e^{-y/2} dy = \lim_{t\to\infty} \int_4^t e^{-y/2} dy$$

and then

$$\int_4^t e^{-y/2} dy = [-2e^{-y/2}]_4^t = 2(e^{-2} - e^{-t/2})$$

which means that

$$\int_4^{\infty} e^{-y/2} dy = \lim_{t\to\infty} \int_4^t e^{-y/2} dy = \lim_{t\to\infty} 2(e^{-2} - e^{-t/2}) = 2e^{-2}$$

Problem 13

$$\int_{-\infty}^{\infty} xe^{-x^2} dx = \int_{-\infty}^{0} xe^{-x^2} dx + \int_0^{\infty} xe^{-x^2} dx = \lim_{t\to\infty} \int_{-t}^{0} xe^{-x^2} dx + \lim_{t\to\infty} \int_0^t xe^{-x^2} dx$$

Notice that

$$\int_0^t xe^{-x^2} dx = [-\frac{1}{2}e^{-x^2}]_0^t = \frac{1}{2}(-e^{-t^2} + 1)$$

therefore

$$\lim_{t\to\infty} \int_0^t xe^{-x^2} dx = \frac{1}{2}$$

and similarly

$$\lim_{t\to\infty} \int_{-t}^{0} xe^{-x^2} dx = -\frac{1}{2}$$

therefore

$$\int_{-\infty}^{\infty} xe^{-x^2} dx = 0$$

which means that the integral is convergent.

Problem 29

$$\int_0^{33} (x-1)^{-1/5} dx = \int_0^1 (x-1)^{-1/5} dx + \int_1^{33} (x-1)^{-1/5} dx =$$

$$\lim_{t\to 1^-} \int_0^t (x-1)^{-1/5} dx + \lim_{t\to 1^1} \int_t^{33} (x-1)^{-1/5} dx$$

Then

$$\int_0^t (x-1)^{-1/5}dx = \frac{5}{4}[(x-1)^{4/5}]_0^t = \frac{5}{4}(x-1)^{4/5} - \frac{5}{4}$$

and

$$\lim_{t \to 1^-} \int_0^t (x-1)^{-1/5}dx = -\frac{5}{4}$$

$$\int_t^{33} (x-1)^{-1/5}dx = \frac{5}{4}[(x-1)^{4/5}]_t^{33} = \frac{5}{4}(32)^{4/5} - \frac{5}{4}(t-1)^{4/5} = 20 - \frac{5}{4}(t-1)^{4/5}$$

and

$$\lim_{t \to 1^1} \int_t^{33} (x-1)^{-1/5}dx = 20$$

Therefore

$$\int_0^{33} (x-1)^{-1/5}dx = 20 - \frac{5}{4} = \frac{75}{4}$$

is convergent.