Math 42: Midterm 2

Formula Sheet

The Binomial Theorem:

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \text{ where } \binom{k}{n} = \frac{k(k-1)(k-2)\dots(k-n+1)}{n!}.$$

Radius of convergence = 1.

Alternating Series Estimation: $|S - S_N| \le b_{N+1}$.

Integral Test Estimation: $|S - S_N| \le \int_N^\infty f(x)\,dx$.

Taylor's Inequality:

$$|f(x) - T_N(x)| \le \frac{M}{(N+1)!} d^{N+1} \text{ for } |x-a| \le d,$$

where $M \ge |f^{(N+1)}(x)|$ on $[a-d, a+d]$.

Trapezoid and Midpoint Estimates: Suppose that $|f''(x)| \le K$ for x in $[a,b]$. Then

$$|\mathrm{Err}_{\mathrm{Trap}}| \le \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |\mathrm{Err}_{\mathrm{Midpt}}| \le \frac{K(b-a)^3}{24n^2}.$$

Simpson's Rule: Let n be even. Set $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$ for $0 \le i \le n$. Then

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right].$$

If $|f^{(4)}(x)| \le K_4$ on $[a,b]$, then the error in the above approximation is at most

$$\frac{K_4(b-a)^5}{180n^4}.$$

Arc Length: If a curve is given by $y = f(x)$ for $a \le x \le b$, then its length is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx.$$

Similarly, if it is given by $x = g(y)$ for $a \le y \le b$, then its length is

$$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy.$$