Math 19: Calculus **Winter 2008** **Instructor: Jennifer Kloke**

Midterm 2

Thursday, 02/28/08

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, *you must show all of your work and justify your answers.* Your answer should be clearly labeled.
- It is OK to leave your work unsimplified.
- If you need extra room, use the back sides of each page. If you need more paper, use the extra at the front of the classroom and staple it to your exam. Include all scratch work with the test. Do not unstaple or detach pages from this exam.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Name: _____

Signature: _____

1. _____ (/6 points)
2. _____ (/8 points)
3. _____ (/4 points)
4. _____ (/24 points)
5. _____ (/5 points)
6. _____ (/8 points)
7. _____ (/5 points)
8. _____ (/40 points)

Total. _____ (/100 points)

1. *(6 points)* Determine whether each statement is true or false. **If the statement is true, cite your reasoning. If it is false, provide an example showing the statement to be false.**

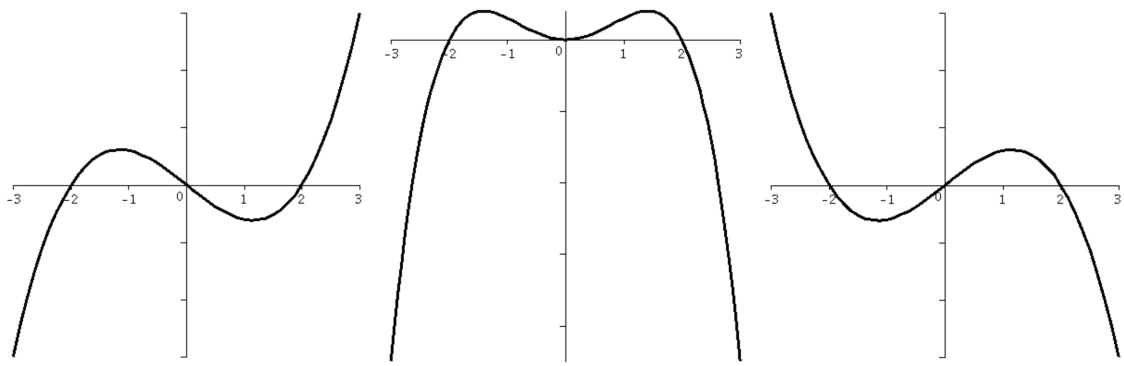
   (a) If a function is differentiable at $x = 0$ it must be continuous at $x = 0$.

   (b) If $f(x)$ satisfies $f'(0) = 0$, then $f(x)$ has a local maximum or a local minimum at $x = 0$.

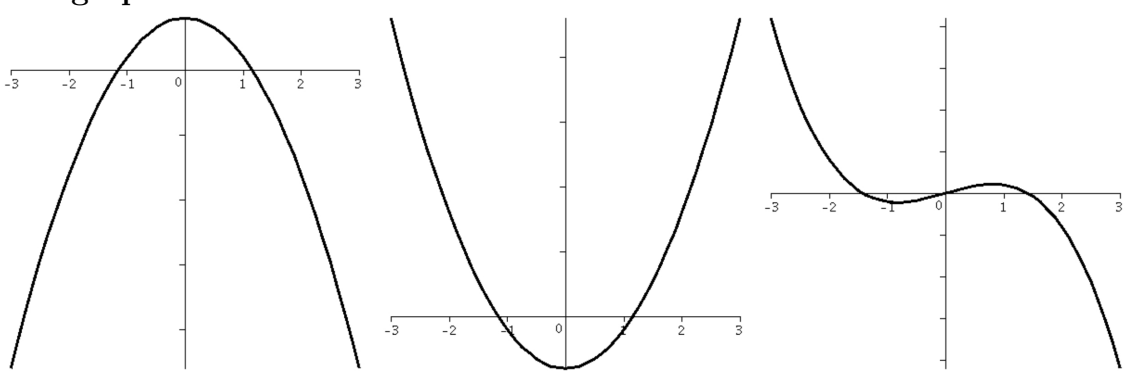
2. *(8 points)* Use the definition of the derivative (i.e. the limit definition) to compute: $\frac{d}{dx}(3x^2 + 15)$.

3. *(4 points)* The graphs of three functions are drawn in the top row. The graphs of their derivatives are drawn in the bottom row. Match each function with its derivative. You do not need to justify your answer, but an incorrect answer without justification will not receive credit.

The graphs of functions.



The graphs of the derivatives of the functions above.



4. Let $f(x) = x^4 + 2x^3 - 89$

(a) *(8 points)* Compute $f'(x)$ and $f''(x)$. You do not have to use the limit definition of the derivative here.

(b) *(4 points)* Find all the critical points of $f(x)$.

(c) *(4 points)* Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

(d) *(4 points)* Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

(e) *(4 points)* From your list of critical points, determine which are actually local maxima and which are actually local minima (for each, be sure to justify why it is a max/min or neither).

5. *(5 points)* Sketch the graph of a single function $f(x)$ with the following properties:

- $f'(0) = 0$,
- $f'(1)$ does not exist,
- $f'(x) > 0$ when $x < -2$ and when $x > 1$,
- $f'(x) < 0$ when $0 < x < 1/2$,
- $f''(x) > 0$ when $x < -1$,
- $f''(x) < 0$ when $-1 < x < 1/2$.

6. *(8 points)* Find the equation of the tangent line to $x^3 - y^3 = -6xy$ at $(3, -3)$.

7. *(5 points)* Suppose $h(x) = 3f(x)g(x) + 2f(x)$, where $f(x)$ and $g(x)$ are differentiable functions. Find $h'(1)$ when $f(1) = 2$, $g(1) = 3$, $f'(1) = 1$, and $g'(1) = -1$.

8. Compute the following derivatives. You do not have to use the definition of the derivative. If you can "do them in your head" instead of showing every step that is up to you (though if you get it wrong we cannot give you partial credit.)

   (a) *(8 points)* Let $f(x) = (7x^3 + 2x)^{90}$. Find $f'(x)$.

   (b) *(8 points)* Let $f(x) = x^8 - 8^x$. Find $f'(x)$.

(c) *(8 points)* Differentiate $(\frac{\tan(x)}{e^x + x})$.

(d) *(8 points)* Find $\frac{d}{dx}(\sqrt[3]{x}(\ln(x)))$.

(e) *(8 points)* Find $\frac{d}{dx}(\cos(x^2)e^{cos(x^2)})$.