

Math 19: Fall 2012
Midterm 2
Version 2

NAME:

LECTURE:

Time: **75 minutes**

This is a closed book and closed notes exam. Calculators and any electronic aid are not allowed.

For each problem, you should write down all of your work carefully and legibly to receive full credit. When asked to justify your answer, you should use theorems and/or mathematical reasoning to support your answer, as appropriate.

I understand and accept the provisions of the Stanford Honor Code.

Signature: _____

Problem	Value	Score
1	13	
2	24	
3	15	
4	25	
5	13	
6	10	
TOTAL	100	

Problem 1 : (13 points)

- a) (5 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Write down the definition of the derivative of f at the point $x = a$, where a is a real number.
- b) (3 points) Suppose that $f: \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at $x = a$. Is f continuous at $x = a$? You may simply answer “yes”, “no”, or “not enough information to decide”.
- c) (5 points) Give an example of a function f and a point $x = a$ such that f is continuous at $x = a$ but f is not differentiable at $x = a$.
If you cannot give such a function, for a maximum of 3 points you may draw the graph of a function f that is continuous but not differentiable at a point. On your graph, label the point at which f is continuous but not differentiable.

Problem 2 : (24 points) For each of the following functions, compute the rule for $f'(x)$. You do not need to simplify your answer.

a) (6 points) $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{4x} \sin x$.

b) (6 points) $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{\cos x}{x^3 + 1}$.

c) (6 points) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = (x + 3)^3(x - 6)^5$.

d) (6 points) $f: [0, 100] \rightarrow \mathbb{R}, f(x) = \sqrt{\sin\left(\frac{\pi}{100}\sqrt{x}\right)}$.

Problem 3 : (15 points) Use the definition of the derivative to compute the derivative of f . No points will be awarded to answers that use derivative rules to compute the derivative.

$$f: \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = 3x^2 - 2x + 3$$

Problem 4 : (25 points) Let f be given by the rule $f(x) = \frac{x+2}{(x-1)^2}$.

a) (2 points) What is the domain of f ?

b) (2 points) Find all of the vertical asymptotes of f .

c) (3 points) Find all of the horizontal asymptotes of f .

d) (3 points) Compute the rule for $f'(x)$.

e) (3 points) List the points at which $f'(x) = 0$ or $f'(x)$ does not exist, if any. If there are none, write “none”.

f) (3 points) For which values of x is $f'(x) > 0$? For which values of x is $f'(x) < 0$? Write your answer in interval notation, and make sure to clearly label which answer is which.

g) (3 points) Compute the rule for $f''(x)$.

h) (3 points) List the points at which $f''(x) = 0$ or $f''(x)$ does not exist, if any. If there are none, write “none”.

i) (3 points) For which values of x is $f''(x) > 0$? For which values of x is $f''(x) < 0$? Write your answer in interval notation, and make sure to clearly label which answer is which.

- j) (5 points) Sketch the graph of f . Label all local maxima and minima and all inflection points, as well as all asymptotes. In particular, if f has extrema or inflection points, you should find the value of f at those points. Your graph should reflect your answers to previous parts of the problem.

Problem 5 : (13 points) Dr Rhoades and Dr Vincent are having a race! Dr Vincent starts running at a velocity of 12 ft/sec, but quickly gets tired so after 3 seconds she slows down to a velocity of 8 ft/sec. A consummate gentleman, Dr Rhoades lets Dr Vincent get a 2 second head start. Afterwards, he runs at a steady pace of 10 ft/sec.

Assume all velocities change instantaneously.

- a) (5 points) Draw the graph of the velocity functions of Dr Rhoades and Dr Vincent on separate graphs. Make sure that it is clear which graph is which by giving each graph a descriptive title.

- b) (5 points) Using the fact that the position function is the antiderivative of the velocity function, draw the graph of the position functions of Dr Rhoades and Dr Vincent on separate graphs. Again, give each graph a descriptive title.

c) (3 points) If they are racing to an object that is 150 ft away, who will win? Explain your reasoning as best as you can, using math as much as possible.

Problem 6 : (10 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f'(x)$ exists for all x . Moreover, assume that f is an even function.

a) (5 points) Use the definition of the derivative to show that $f'(-x) = -f'(x)$ for all x .

b) (2 points) What mathematical word could you use to describe the function f' ?

c) (3 points) What is the value of $f'(0)$? Justify your answer briefly.