MATH 19
MIDTERM 1: FEBRUARY 6, 2012

Instructions.

(1) Complete the following problems. Unless otherwise stated, you may use any results from class or from Sections 1.1 - 2.3 of the book that you like. If you use a result, be sure to verify the hypotheses are satisfied.
(2) this is a closed-book exam. No calculators or other electronic aids will be permitted.
(3) In order to receive full credit you must show all your work and justify you answers.
(4) Please sign the honor code statement on the blue book that includes your answers.

Basic Limits

(1) Let a and c be a real numbers and $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = c$, then $\lim_{x\to a} f(x) = c$.
(2) Let a be a real number and $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x$, then $\lim_{x\to a} f(x) = a$.
(3) Let $f : D \to [0, \infty)$ be a function and $a \in D$, then $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ provided that $\lim_{x\to a} f(x)$ exists.

Basic Limit Laws Suppose that c is a constant and the limits

$$\lim_{x\to a} f(x) \quad \text{and} \quad \lim_{x\to a} g(x)$$

exist. Then

(1) $\lim_{x\to a} (f(x) + g(x)) = (\lim_{x\to a} f(x)) + (\lim_{x\to a} g(x))$
(2) $\lim_{x\to a} (f(x) - g(x)) = (\lim_{x\to a} f(x)) - (\lim_{x\to a} g(x))$
(3) $\lim_{x\to a} cf(x) = c \cdot (\lim_{x\to a} f(x))$.
(4) $\lim_{x\to a} (f(x) \cdot g(x)) = (\lim_{x\to a} f(x)) \cdot (\lim_{x\to a} g(x))$
(5) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided that $\lim_{x\to a} g(x) \neq 0$.

Date: February 6, 2012.

Problem 1. Define a function.

Problem 2.

(1) Give an example of an even function. Explain why it is even.
(2) Draw the graph of a function which is neither even nor odd. Justify your answer.
(3) Give an example of a function which is increasing on the interval $[0, 2]$. Explain why it is increasing.

Problem 3. Use only the Basic Limits and Basic Limit Laws (given above) to calculate the following limits.

(1)
$$\lim_{x \to 0} \frac{37}{x^5 + x^4 + 2x^3 + 3x^2 + 5x + 8}.$$
(2)
$$\lim_{x \to 0} \sqrt[5]{\frac{x^2 + 64}{x^4 + 2}}$$

Problem 4. Calculate the following limits. Justify all of your steps.

(1)
$$\lim_{x \to -2} \frac{x^3 + 3x^2 + 2x}{x^3 + 5x^2 + 6x}$$
(2) Let $f : [0, 2] \to \mathbb{R}$. Assume that $3x \leq f(x) \leq x^3 + 2$ for all x in the domain of f. Calculate
$$\lim_{x \to 1} f(x).$$

Problem 5.

(1) Define what it means for a function to be continuous at a number 2.
(2) Give an example of a function which is not continuous at π. Justify your answer.

Problem 6.

(1) If $f : \mathbb{R} \to \mathbb{R}$ is a function given by $f(x) = 2^{3x} + 2^{2x} + x + 1$ find $f^{-1}(3)$ and $f(f^{-1}(1))$.
(2) Let $f : (-\infty, 0) \to (0, \infty)$ be given by $f(x) = x^2$. Compute f^{-1}, state its domain, and show that $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ fo all appropriate x.