**Math 19: Calculus** **Winter 2008** **Instructor: Jennifer Kloke**

# Midterm 1

Thursday, 01/31/08

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, *you must show all of your work and justify your answers.* Your answer should be clearly labeled.
- It is OK to leave your work unsimplified.
- If you need extra room, use the back sides of each page. If you need more paper, use the extra at the front of the classroom and staple it to your exam. Include all scratch work with the test. Do not unstaple or detach pages from this exam.
- Please sign the following:

  > "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Name: _____

Signature: _____

1. _____ (/12 points)
2. _____ (/16 points)
3. _____ (/6 points)
4. _____ (/20 points)
5. _____ (/20 points)
6. _____ (/10 points)
7. _____ (/5 points)
8. _____ (/6 points)
9. _____ (/5 points)

Total. _____ (/100 points)

1. *(12 points)* Give the MATHEMATICAL definitions of

   (a) The function $f(x)$ is continuous on the interval [a,b].

   (b) The tangent line to the curve $y = f(x)$ at the point $P = (a, f(a))$.

   (c) The limit of the function $f(x)$, as $x$ goes to $a$, equals $L$.

2. Show all your reasoning in the problems below. Reference any limit laws used. Let

$$g(x) = \begin{cases} x & : \ x \leq -\pi \\ \sin(x) & : \ -\pi < x < \pi \\ 1 & : \ x = \pi \\ 2x - 2\pi & : \ x > \pi. \end{cases}$$

(a) *(4 points)* Graph $g(x)$. Please draw the graph as accurately as possible and with labels.

(b) *(3 points)* Find $\lim_{x \to (-\pi)^-} g(x)$.

(c) *(3 points)* Find $\lim_{x \to (-\pi)^+} g(x)$.

(d) *(4 points)* Find $\lim_{x \to \pi} g(x)$.

(e) *(2 points)* Is $g(x)$ continuous at $x = \pi$?

3. *(6 points)* Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide an example showing the statement to be false.**

   (a) If $f(0)$ is undefined, then $\lim_{x \to 0} f(x)$ does not exist.

   (b) If $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ do not exist, then $\lim_{x \to 0}(f(x) - g(x))$ does not exists.

4. Compute the following limits; justify your answers. You are allowed to use any rules we've shown in class; quote the rules you use. If a limit does not exist, explain why.

(a) *(5 points)* $\lim_{x \to (-1)} \dfrac{x^4 - 3x^2 - 8}{x^2 - \sqrt{4x + 8}}$

(b) *(5 points)* $\lim_{x \to (-2)} \dfrac{x^2 - 3x - 10}{x^2 + x - 2}$

(c) *(5 points)* $\lim_{x \to 0} (\tan(x^2) + e^x)^2$

(d) *(5 points)* $\lim_{h \to 0} \dfrac{h + \sqrt{4+h} - 2}{h}$

5. Let

$$f(x) = 3x^2 + 6.$$

(a) *(10 points)* Using the limit definition of the derivative at a point, compute $f'(a)$.

(b) *(5 points)* Find the equation of the tangent line to the curve at $x = 1$.

(c) *(5 points)* Suppose that $f(x)$ represents the position of a particle at time $x$.

i. Explain, in words, what the slope of the secant line between the points (0,6) and (2,18) means.

ii. Explain, in words, what the slope of the tangent line to the curve at the point (2,18) means.

6. *(10 points)* Let $f(x) = \dfrac{x^2}{(x+1)}$. Find $f'(1)$ using the limit definition of the derivative. Show the steps of your computation.

7. *(5 points)* Prove that

$$f(x) = \begin{cases} \cos(x) & : \quad x \leq 0 \\ e^{\sqrt{x}} & : \quad x > 0 \end{cases}$$

is continuous on $(-\infty, \infty)$. Show all your reasoning.

8. *(6 points)* Jennifer's position at time $t$ relative to Stone Cold Creamery is given by the function

$$p(t) = 5 + 2t - t^2$$

(positive indicates west and negative east). Explain why Jennifer must have visited the creamery between the hours of $t = 0$ and $t = 4$. Justify your answer with results from class!

9. *(5 points)* Sketch the graph of a *continuous* function that has the following properties:

- $f(0) = 1$
- $f(1) = 0$
- $f'(-1) = -2$
- $f'(0) = 1$
- $f'(1) = 0$.

Please put labels on your axes.