Midterm 1

Thursday, 10/15/09

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, *you must show all of your work and justify your answers.* Your answer should be clearly labeled.
- If you need extra room, use the back sides of each page. Staple any scratch paper to your exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.

Name: _____

Signature: _____

Name: _____

Please leave the following table blank for the grader.

1. _____ (/10 points)
2. _____ (/35 points)
3. _____ (/10 points)
4. _____ (/10 points)
5. _____ (/5 points)
6. _____ (/10 points)
7. _____ (/10 points)
8. _____ (/10 points)

Total. _____ (/100 points)

1. (10 points) The following question is about the mathematical definition of certain concepts.

   (a) (2 points) State the definition of one-to-one function.

   (b) (2 points) Given a function $y = f(x)$, state the definition of continuity at the point $x = a$.

   (c) (2 points) Given a function $y = f(x)$, state the definition of the derivative at the point $x = a$, i.e. define $f'(a)$.

   (d) (2 points) For a function $y = f(x)$, what does $f'(2)$ represent geometrically?

   (e) (2 points) State the Intermediate value theorem.

2. (35 points) Compute the limit. Show your work! If a limit does not exist, show why.

(a)

$$\lim_{x \to -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3}.$$

(b)

$$\lim_{x \to 1} \frac{x^3 - 2x^2 - x + 2}{x^3 - x^2}.$$

(c)

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right).$$

(d)

$$\lim_{x\to 1}\left(e^{2x}-\ln(x)+x^2+2x+1+\sin(\pi x)\right).$$

(e)

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}.$$

(f)

$$\lim_{x \to 1} e^{x^3 - x}.$$

(g)

$$\lim_{t \to 1} \frac{|t-1|}{t-1}.$$

3. (10 points) Consider the piecewise defined function defined below

$$g(x) = \begin{cases} x+2, & x < -2 \\ x^2, & -2 \leq x < 1 \\ 0, & x = 1 \\ -x+2, & x > 1 \end{cases}$$

(a) (4 points) Sketch the graph of $g(x)$.

(b) (2 points) What is $\lim_{x \to -2} g(x)$?

(c) (2 points) What is $\lim_{x \to 1} g(x)$?

(d) (2 points) Is $g(x)$ continuous at $x = 1$? Explain graphically and using the definition.

4. (10 points) Consider the function $y = f(x) = \frac{e^x+1}{e^x-1}$.

(a) (2 points) What is the domain of f?

(b) (6 points) Compute the inverse of f^{-1}.

(c) (2 points) What is the range of f?

5. (5 points) If the equation of the tangent line to a curve $y = f(x)$ at the point where $a = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

6. (10 points) Find the equation of the tangent line to $f(x) = x^2 - 2x + 2$ at $x = 2$ using the limit definition of the derivative (i.e. you may not use any differentiation rules you may have learned about in previous calculus courses!)

7. Prove that there is a real-valued solution to the equation

$$\sqrt{x-5} = \frac{1}{x+3}.$$

8. (10 points) Determine if the following statements are true or false for arbitrary functions. If the statement is true, cite your reasoning. If the statement is false, provide an example showing it is false.

   (a) (2 points) If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ does not exist.

   (b) (2 points) The graph of a one-to-one function passes the vertical line test.

   (c) (2 points) If f and g are functions then $f \circ g = g \circ f$.

   (d) (2 points) If f is continuous on $[-1, 1]$ and $f(1) = 0$ and $f(-1) = 0$ then there exists a value c where $-1 < c < 1$ where $f(c) = 0$.

   (e) (2 points) $f(x + y) = f(x) + f(y)$, for any real numbers x and y.