Math 19 Practice Exam 2A, Winter 2011

Name: _____ **SUID#:** _____

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers.
- You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- **You have 2 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
- Please check that your copy of this exam contains 10 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: _____

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	8	**Total**
Points:	10	8	14	12	14	13	12	7	90
Score:									

1. (10 points) Find each of the following limits, with justification (show steps). If there is an infinite limit, then explain whether it is ∞ or $-\infty$.

   (a) (5 points) $\lim_{t \to 0} \dfrac{\sqrt{1+t} - (t+1)}{t}$

   (b) (5 points) $\lim_{x \to 3^+} \dfrac{x^2 - 4x + 3}{4x^2 - 13x + 3}$

2. (8 points) Use the definition of the derivative as a limit to find the derivative of $f(x) = \dfrac{-2}{1-x}$. No marks will be awarded if you get the right answer using other methods.

3. (14 points) A function $f(x)$ satisfies the following conditions:

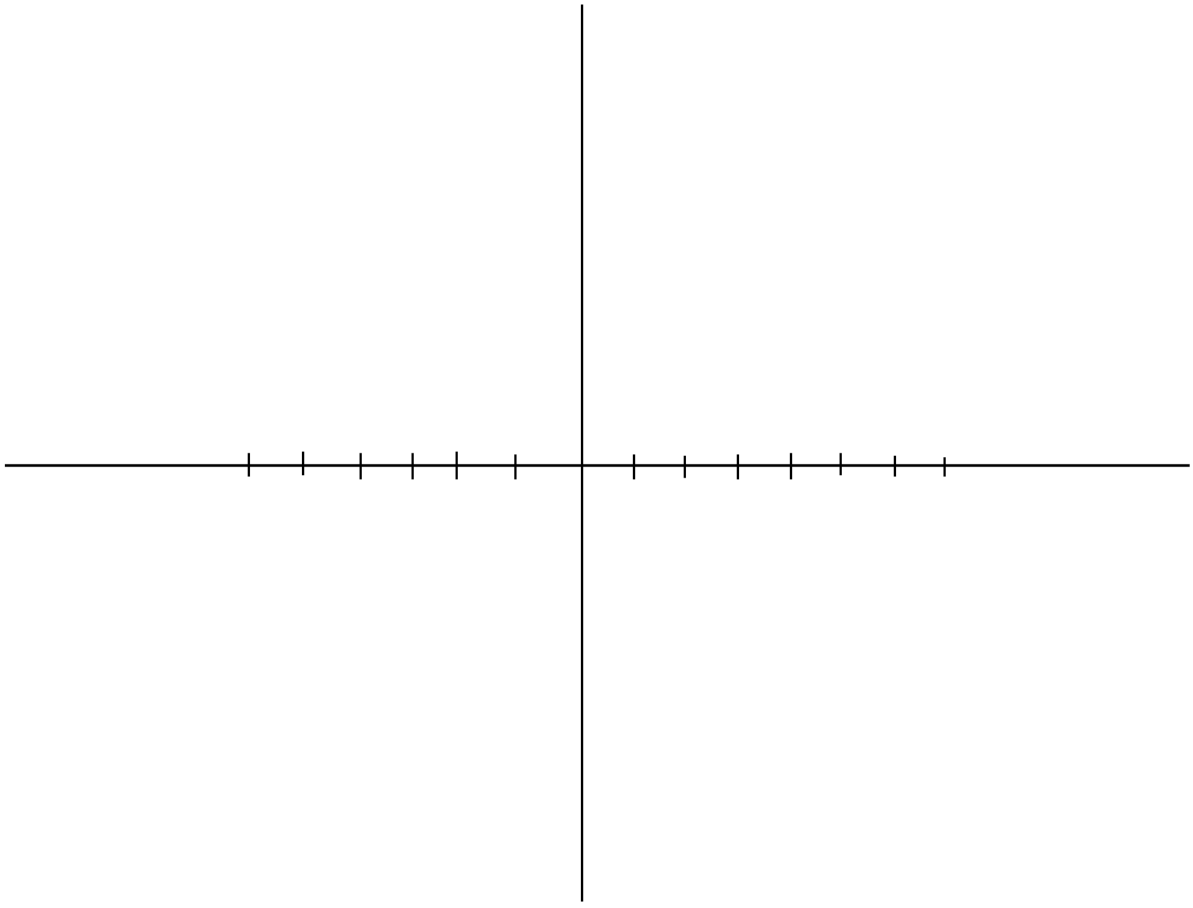
   - $f'(x) > 0$ if $0 < x < 3$
   - $f'(x) < 0$ if $x < 0$ or $x > 3$
   - $f'(0) = f'(3) = 0$
   - $f''(x) < 0$ if $x < -1$ or $x > 2$
   - $f''(x) > 0$ if $-1 < x < 2$
   - $f''(-1) = f''(2) = 0$

   (a) (3 points) Where is $f(x)$ increasing? And decreasing? Use the interval notation for your answer.

   (b) (3 points) Where is $f(x)$ concave up? And concave down? Use the interval notation for your answer.

   (c) (3 points) Find the x-coordinates of all local maxima, local minima and inflection points.

(d) (5 points) Sketch the graph of $f(x)$ in the axes provided below.



4. (12 points) Find the derivatives of the following functions using any method you like. You do not need to simplify your answers.

   (a) (4 points) $f(x) = \dfrac{x + x^2 + x^3}{\sqrt{x}}$

   (b) (4 points) $g(x) = \dfrac{\sin(x)\ln(\ln(x))}{\cos(x)}$

   (c) (4 points) $Z(t) = 3^{t^2+5t}$

5. (14 points) Careful: These two problems are unrelated.

   (a) (6 points) Find the tangent line to $f(x) = e^{\cos(x)+\sin(x)}$ at $x = \pi$.

   (b) (8 points) Find the derivative of $g(x) = (\sqrt{x})^{\ln(x+3)}$.

6. (13 points) Given the function $G(x) = \begin{cases} \frac{2}{x} & \text{if } x \leq 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$

(a) (5 points) Is $G(x)$ continuous at $x = 1$? Justify your answer completely.

(b) (8 points) Is $G(x)$ differentiable at $x = 1$? Justify your answer completely.

7. (12 points) Find the derivatives of the following functions using any method you like. You do not need to simplify your answers.

(a) (4 points) $f(x) = \cos(e^{\arctan(x)})$

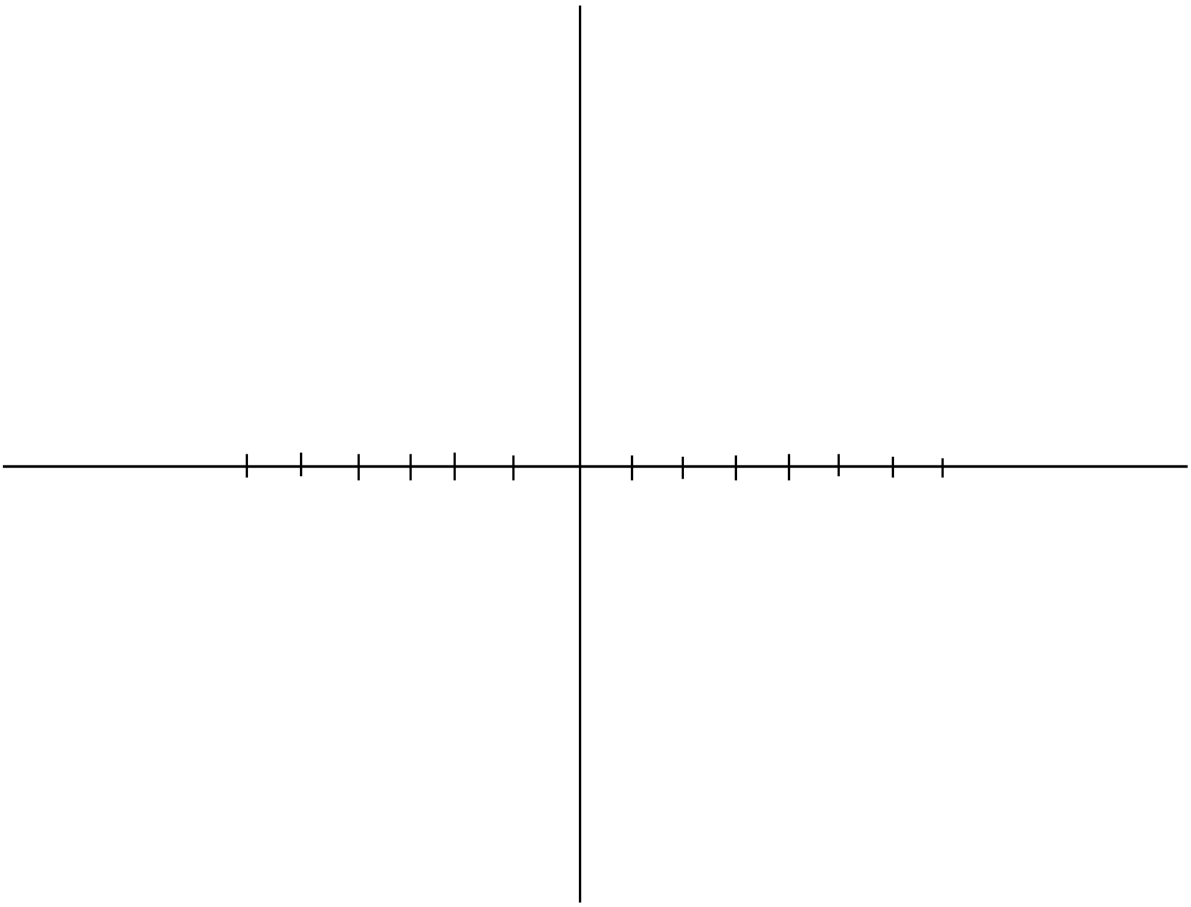
(b) (4 points) $g(x) = \dfrac{\arcsin(x)}{x^6 + 5x^4}$

(c) (4 points) $Z(t) = 6\tan(t) - \dfrac{4}{t^7} + 3t^5$

8. (7 points) Careful: These two problems are unrelated.

   (a) (3 points) State the intermediate value theorem.

   (b) (4 points) In the axes provided below, draw the graph of a function that satisfies $f'(-1) = 0$, $f'(3) = 0$, $f'(-5) > 0$ and $f'(4) < 0$



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