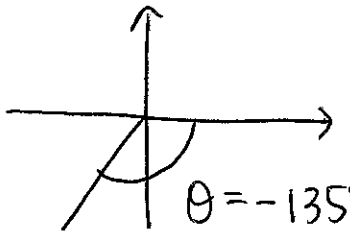
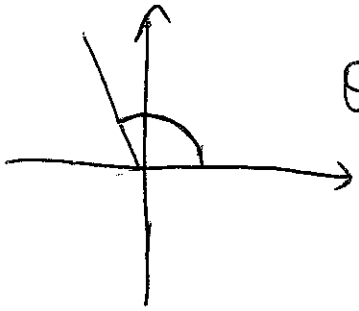
Homework 1 Trigonometry solutions

#1 a)



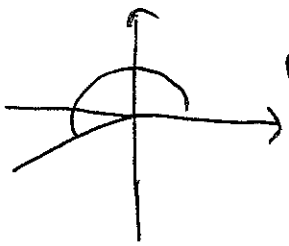
$\theta = -135° = -\frac{3\pi}{4}$ OR $\frac{5\pi}{4}$ radians

b)



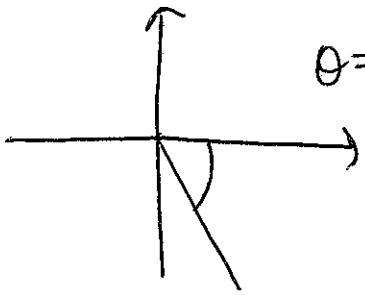
$\theta = 120° = \frac{2\pi}{3}$ radians

c)



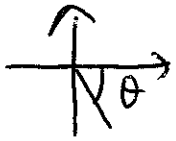
$\theta = \frac{7\pi}{6}$ rad $= 210°$

d)

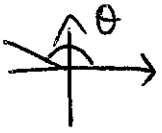


$\theta = \frac{-\pi}{3}$ rad $= -60°$ OR $300°$

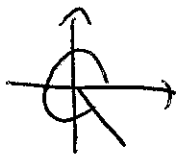
#2 a) $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$



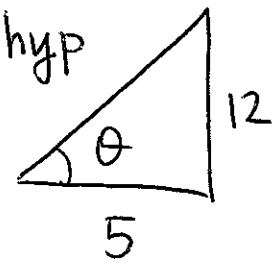
b) $\cos\left(\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2}$



c) $\tan\left(\frac{7\pi}{4}\right) = -1$



#3



$$\text{hyp} = \sqrt{12^2+5^2}$$
$$= \sqrt{144+25}$$
$$= \sqrt{169} = 13$$

$$\sin\theta = \frac{12}{13}, \quad \cos\theta = \frac{5}{13}, \quad \csc\theta = \frac{13}{12}$$

$$\sec\theta = \frac{13}{5}, \quad \cot\theta = \frac{5}{12}$$

#4 a) $\sin^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) = 1$

b) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

c) $\sin^2\frac{\pi}{8} = \frac{1-\cos\frac{2\pi}{8}}{2}$

$= \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2} = \frac{2-\sqrt{2}}{4}$

#5 $\sin^4(x) = (\sin^2 x)^2$

$= \left(\frac{1-\cos 2x}{2}\right)^2$

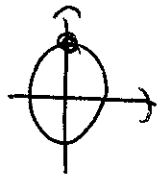
$= \frac{1-2\cos 2x+\cos^2 2x}{4}$

$= \frac{1-2\cos 2x+\frac{1+\cos 4x}{2}}{4}$

$= \frac{\cos 4x-4\cos 2x+3}{8}$

#6 a) $\sin x = 1$ when $x = \frac{\pi}{2} + 2n\pi$

for n any integer



b) let $a = \sin x$ then

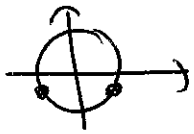
$2\sin^2 x - \sin x - 1 = 0$ is $2a^2 - a - 1 = 0$

$(2a+1)(a-1) = 0$

$2a+1=0$ $\quad$ $a-1=0$

$a = -\frac{1}{2}$ $\quad$ $a = 1$

So either $\sin x = -\frac{1}{2}$



and $x = \frac{7\pi}{6} + 2n\pi$, n an integer

OR $x = \frac{11\pi}{6} + 2n\pi$, n an integer

OR $\sin x = 1$ and $x = \frac{\pi}{2} + 2n\pi$, n an integer

so all the solutions are

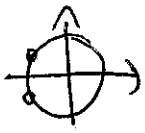
$x = \frac{7\pi}{6} + 2n\pi$, $x = \frac{11\pi}{6} + 2n\pi$, $x = \frac{\pi}{2} + 2n\pi$

where each time n is an integer

c) $(2\cos x + \sqrt{3})(2\sin x - 1) = 0$

$2\cos x + \sqrt{3} = 0$ $\qquad$ $2\sin x - 1 = 0$

$\cos x = \frac{-\sqrt{3}}{2}$ $\qquad$ $\sin x = \frac{1}{2}$



$x = \frac{5\pi}{6} + 2n\pi$ $\qquad$ $x = \frac{5\pi}{6} + 2n\pi$

$x = \frac{7\pi}{6} + 2n\pi$ $\qquad$ $x = \frac{\pi}{6} + 2n\pi$

where n is an integer