

Calculus Pre-test

Name:
SUNet ID:

SOLUTIONS

This pre-test is intended to help you gauge your preparedness for Stanford math classes. If you feel that you did not do as well on these problems as you would like, you have a few options. One is that you work on your own or with friends to better familiarize yourself with this material. The other is to take advantage of the precalculus resources offered by the Math Department. You can find out more about the Stanford Precalculus Resource Portal at <http://precalculus.stanford.edu>.

This pre-test is not for credit, and the score you receive will be in no way part of your record for this course. Please do your best, but do not hesitate to leave a question blank if you do not know the answer.

Question 1: Give the equation of the line parallel to the line $y = 5x - 6$ and passing through the point $(-10, 3)$.

A) $y = -\frac{1}{5}x + 1$

B) $y = \frac{1}{5}x + 5$

C) $y = -\frac{1}{5}x - \frac{47}{5}$

D) $y = 5x + 53$

The line $y = 5x - 6$ has slope 5, and a line parallel to this line also has slope 5.

The line we are looking for has equation

$y = 5x + b$. To solve for b , plug in $(x, y) = (-10, 3)$:

$$3 = 5(-10) + b$$

$$3 = -50 + b$$

$$b = 53$$

$$\text{so } y = 5x + 53$$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 2: Simplify the following expression completely:

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{1}{5}}$$

A) $\frac{55}{120}$

B) $\frac{50}{33}$

C) $-\frac{1}{5}$

D) $\frac{1}{5}$

Solution 1: $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{1}{5}} = \frac{\frac{3}{6} + \frac{2}{6}}{\frac{15}{20} - \frac{4}{20}} = \frac{5}{6} \div \frac{11}{20} = \frac{5}{6} \cdot \frac{20}{11} = \frac{5 \cdot 10}{3 \cdot 11} = \frac{50}{33}$

Solution 2: $\frac{(\frac{1}{2} + \frac{1}{3}) 60}{(\frac{3}{4} - \frac{1}{5}) 60} = \frac{30 + 20}{45 - 12} = \frac{50}{33}$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 3: Simplify the following expression completely:

$$\frac{\sqrt{x \sqrt[3]{y}}}{x^{-2}}$$

A) $x^{5/2}y^{1/6}$

B) $x^{-3/2}y^{1/3}$

C) $x^{5/2}y^{1/3}$

D) $x^{-3/5}y^{1/5}$

$$\frac{\sqrt{x \sqrt[3]{y}}}{x^{-2}} = \frac{(x y^{1/3})^{1/2}}{x^{-2}} = \frac{x^{1/2} y^{1/6}}{x^{-2}}$$

$$= x^{1/2} x^2 y^{1/6}$$
$$= x^{5/2} x^{4/2} y^{1/6} = x^{5/2} y^{1/6}$$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 4: Factor the following expression completely:

$$x^4 - y^4.$$

A) $(x^2 - y^2)(x^2 + y^2)$

B) $(x^2 - y^2)(x + y)(x - y)$

C) $(x^2 + y^2)(x + y)(x - y)$

D) $(x^2 - y^2)(x^2 + y^2)(x + y)(x - y)$

$$\begin{aligned}x^4 - y^4 &= (x^2 - y^2)(x^2 + y^2) \\ &= (x - y)(x + y)(x^2 + y^2)\end{aligned}$$

Note: $(x^2 - y^2)(x^2 + y^2)$ is correct but
not factored completely

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 5: Simplify the following rational expression completely:

$$\frac{3x^2 - 2x - 1}{3x^2 - 14x - 5}$$

A) $\frac{x+1}{x+5}$

B) $\frac{x-1}{x-5}$

C) $x^2 + 7x + 5$

D) The expression is already in simplest form.

$$\begin{aligned} 3x^2 - 2x - 1 &= 3x^2 - 3x + x - 1 = 3x(x-1) + 1(x-1) \\ &= (3x+1)(x-1) \end{aligned}$$

$$\begin{aligned} 3x^2 - 14x - 5 &= 3x^2 - 15x + x - 5 = 3x(x-5) + 1(x-5) \\ &= (3x+1)(x-5) \end{aligned}$$

$$\frac{3x^2 - 2x - 1}{3x^2 - 14x - 5} = \frac{(3x+1)(x-1)}{(3x+1)(x-5)} = \frac{x-1}{x-5}$$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 6: Complete the square:

$$x^2 - 4x + 1.$$

- (A) $(x-2)^2 - 3$ B) $(x+2)^2 + 3$ C) $(x-4)^2 - 15$ D) $(x+2)^2 - 3$

$$x^2 - 4x + 4 = (x-2)^2$$

So
$$\begin{aligned} x^2 - 4x + 1 &= x^2 - 4x + 4 - 4 + 1 \\ &= (x-2)^2 - 3 \end{aligned}$$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

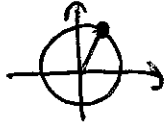
Question 7: What is $\sin\left(\frac{\pi}{3}\right)$? (Here the measure of the angle is given in radians.)

A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{3}$



How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 8: Consider the parabola whose graph is given by the equation

$$y = 6x^2 - x - 1.$$

What are the x - and y -intercepts of this graph?

A) The x -intercept is $(0, -1)$ and the y -intercepts are $(1/2, 0)$ and $(-1/3, 0)$.

B) The x -intercepts are $(-1/2, 0)$ and $(1/3, 0)$ and the y -intercept is $(0, -1)$.

C) The x -intercept is $(0, -1)$ and the y -intercepts are $(-1/2, 0)$ and $(1/3, 0)$.

D) The x -intercepts are $(1/2, 0)$ and $(-1/3, 0)$ and the y -intercept is $(0, -1)$.

y -int: when $x=0 \rightsquigarrow y = 6 \cdot 0^2 - 0 - 1 = -1 \quad (0, -1)$

x -int: when $y=0 \rightsquigarrow 0 = 6x^2 - x - 1$
 $= 6x^2 - 3x + 2x - 1 \quad (-\frac{1}{3}, 0)$
 $= 3x(2x - 1) + 1(2x - 1) \quad (\frac{1}{2}, 0)$
 $= (3x + 1)(2x - 1) \quad x = -\frac{1}{3}, \frac{1}{2}$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 9: Consider the function given by the rule

$$f(x) = \sqrt{6-x}$$

What is the domain of this function?

A) $(-\infty, \infty)$

B) $(6, \infty)$

C) $[6, \infty)$

D) $(-\infty, 6]$

Must have $6-x \geq 0$ to take square root
 $6 \geq x$

In interval notation this is $(-\infty, 6]$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)

Question 10: Simplify the following expression completely:

$$\log(x+3) - \log(x-2) + \log(x^2+4).$$

A) $\log[(x+3)(x-2)(x^2+4)]$

C) $\log\left[\frac{(x+3)(x^2+4)}{(x-2)}\right]$

B) $\log[(x+3) - (x-2) + (x^2+4)]$

D) $\log\left[\frac{(x+3)}{(x-2)(x^2+4)}\right]$

How well do you understand the concepts in this problem and how easily could you solve it?

1

2

3

4

5

6

7

(not at all)

(extremely well)