

Math 19: Winter 2015  
Midterm 1

NAME:

Time: **50 minutes**

For each problem, you should write down all of your work carefully and legibly to receive full credit. When asked to justify your answer, you should use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason.
- You are bound by the Stanford Honor Code, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: \_\_\_\_\_

Problem	Value	Points
1	15	
2	20	
3	16	
4	18	
5	8	
6	8	
7	15	

1. (True/False) Determine whether each of the following statements is true or false. You do **not** need to justify your answer. (3 pts. each)

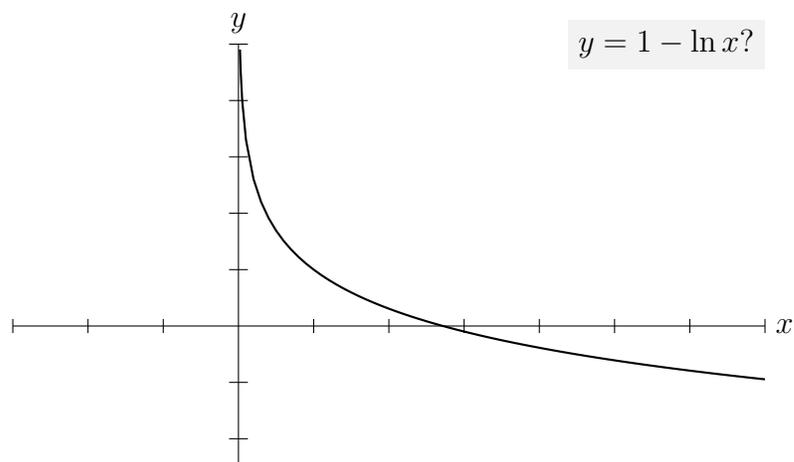
a. (True or false) The function  $f(x) = x^{2n}$  is one-to-one, where  $n$  is an integer.

b. (True or false) The function  $f(x) = x^{2n+1}$  is one-to-one, where  $n$  is an integer.

c. (True or false) The function  $\sin^{-1} x$  is the inverse function of  $\sin x$ , so  $\sin^{-1}(\sin x) = x$  for all  $x$ .

d. (True or false) The function  $\arcsin x$  is the inverse function of  $\sin x$ , so  $\sin(\arcsin x) = x$  for all  $x$ .

e. (True or false) The following is the graph of  $y = 1 - \ln x$ .



2. Consider the function  $f(x) = \frac{x-2}{x-1}$ .
- (3 pts.) What is the domain of  $f$ ?
  - (5 pts.) Find a formula for  $f^{-1}$ .
  - (4 pts.) What is the domain of  $f^{-1}$ ? What does this say about the range of  $f$ ?
  - (3 pts.) Does  $f(x)$  have any vertical asymptotes? If so, what are their equations?
  - (5 pts.) Does  $f(x)$  have any horizontal asymptotes? If so, what are their equations?

**3.** In each of the following problems, evaluate the limit. If it does not exist, explain why. If it is an infinite limit, determine whether it's  $+\infty$ ,  $-\infty$ , or neither.

**a.** (8 pts.)  $\lim_{x \rightarrow -\infty} \frac{x^5 - 12x^3 + 1}{5x^4 + 2x + 1}$

**b.** (8 pts.)  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{|x - 2|}$

**4. a.** (10 pts.) Find the values of  $a$  and  $b$  for which the function  $f(x)$  is continuous, where

$$f(x) := \begin{cases} x^2 - 2, & x \leq -1 \\ ax + b, & -1 < x \leq 1 \\ 4 - x^2, & x > 1. \end{cases}$$

**b.** (8 pts.) Sketch the graph of the function  $f(x)$  with your values of  $a$  and  $b$  (or leave that part of the graph empty, if you haven't done **a.**). Explain what it means in terms of the graph to choose  $a$  and  $b$  so that  $f(x)$  is continuous.

6

5. (8 pts.) Prove that the polynomial  $f(x) = x^8 - 5x^3 + 2$  has a root in the interval  $[-1, 1]$ .

6. (8 pts.) Using the squeeze theorem, find  $\lim_{x \rightarrow 0} x^4 \cos \frac{\pi}{x}$ .

**7. a.** (10 pts.) Using the **limit definition**, find the derivative of the function  $f(x) = \sqrt{4-x}$ .

**b.** (5 pts.) Find the equation for the tangent line to the curve  $y = \sqrt{4-x}$  at the point  $(3, 1)$ .