Exponentials & logarithms practice Solutions

Sec 1.5 #2 a) $8^{4/3} = (8^{1/3})^4 = 2^4 = 16$

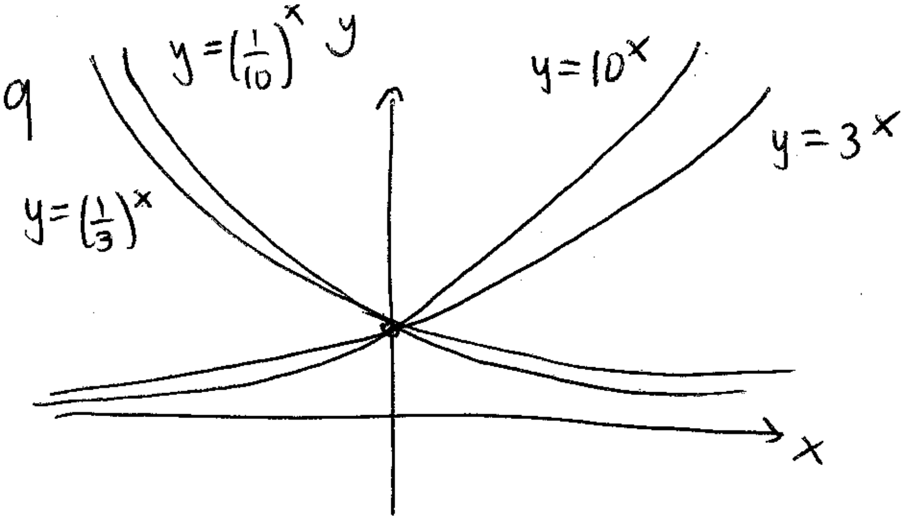
b) $x(3x^2)^3 = x \cdot 3x^2 \cdot 3x^2 \cdot 3x^2 = 27x^7$

#4 a) $\dfrac{x^{2n} \cdot x^{3n}}{x^{n+2}}$

b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{(a b^{\frac{1}{2}})^{\frac{1}{2}}}{(ab)^{\frac{1}{3}}} = \dfrac{a^{\frac{1}{2}} b^{\frac{1}{4}}}{a^{\frac{1}{3}} b^{\frac{1}{3}}}$

$= \dfrac{a^{\frac{3}{6}} \; b^{\frac{3}{12}}}{a^{\frac{2}{6}} \; b^{\frac{4}{12}}} = \dfrac{a^{1/6}}{b^{1/12}}$

#9



Section 1.6 #37 a) $\log_2 6 - \log_2 15 + \log_2 20$

$$= \log_2 \left(\frac{\cancel{6}^{2} \cdot \cancel{20}^{4}}{\cancel{15}\,\cancel{5}_{1}} \right)$$

$$= \log_2 8 = \log_2 2^3 = 3$$

b) $\log_3 100 - \log_3 18 - \log_3 50$

$$= \log_3 \left(\frac{\cancel{100}^{\cancel{2}\,1}}{\cancel{50} \cdot \cancel{18}\,9} \right)$$

$$= \log_3 \left(\frac{1}{9} \right) = \log_3 3^{-2} = -2$$

#1 a) $\log_b \left(\frac{b^8 x^2}{y^3} \right) = \log_b b^8 + \log_b x^2 - \log_b y^3$

$$= 8 + 2\log_b x - 3\log_b y$$

$$= 8 + 2 \cdot 2.3 - 3 \cdot 3.1$$

$$= 8 + 4.6 - 9.3 = 12.6 - 9.3$$

$$= 3.3$$

b) $\ln\left(\frac{(x^2+4)^5 \sqrt[3]{4x-3}}{\sqrt{3x-5}\,(7x-2)^9}\right)$

$= \ln(x^2+4)^5 + \ln(4x-3)^{\frac{1}{3}}$
$\quad - \ln(3x-5)^{\frac{1}{2}} - \ln(7x-2)^9$

$= 5\ln(x^2+4) + \frac{1}{3}\ln(4x-3)$
$\quad - \frac{1}{2}\ln(3x-5) - 9\ln(7x-2)$

#2 a) $e^{2x} - e^{4x} + 2 = 0$

Let $y = e^{2x}$. Then $y^2 = (e^{2x})^2 = e^{4x}$

so $y - y^2 + 2 = 0$

$y^2 - y - 2 = (y-2)(y+1) = 0$

so $y = 2$ OR $y = -1$

but $y = e^{2x}$ so

$e^{2x} = 2 \rightsquigarrow \ln(e^{2x}) = \ln 2$

$2x = \ln 2$

$\boxed{x = \frac{1}{2}\ln 2}$

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$e^{2x} = -1$ has no solution since $e^x > 0$.

b) $(\ln x)^2 = \ln(x^4)$

We have $\ln(x^4) = 4\ln x$

Let $y = \ln x$

$y^2 = 4y$

$y^2 - 4y = y(y-4) = 0$

so $y = 0$, $y = 4$

But $y = \ln x$

$0 = \ln x \rightsquigarrow e^0 = e^{\ln x}$

$\boxed{1 = x}$

$4 = \ln x \rightarrow e^4 = e^{\ln x}$

$\boxed{4 = x}$