Math 108: Midterm Review

Format: The exam will be 50 minutes, in class, closed notes. A large chunk of the exam will be computational (e.g., how many automorphisms does this graph have?) and applications of theorems and definitions (e.g., does this graph have a certain property?). There will be one or two relatively friendly proofs, and you may be asked to state some of the important theorems.

Topics:

- Basic definitions: Graphs, trees, simple graphs, digraphs, components, isomorphisms, automorphisms, valency, subgraphs, paths, walks, spanning trees, bipartite graphs, forests, weighted graphs, vertex colorings, proper colorings, girth, matchings, etc.
- Eulerian and Hamiltonian circuits. Euler's theorem for graphs and digraphs.
- $\sum_{y \in V(G)} \deg(v) = 2|E(G)|$
- Problem 1C: Trees have $n-1$ edges, etc.
- Cayley's theorem and Prüfer codes
- The number of spanning trees on K_n isomorphic to a given T
- The cheapest spanning tree algorithm
- The statement of Brooks's theorem
- The statement of the four color theorem
- The chromatic polynomial and its meaning, chromatic recurrence: $\chi(G,k) = \chi(G - e, k) - \chi(G \cdot e, k)$
- Basic properties of $\chi(G,k)$, including $\chi(G,k) = k^n - Ek^{n-1} + \dots$, and its relation to $\chi(G)$
- The meaning of the Ramsey numbers $N(p,q) = N(p,q;2)$, $N(p_1,\dots,p_s;2)$, and $N(p_1,\dots,p_s;t)$
- The idea of Turán's theorem, Theorem 4.1; where does the number $M(n,p)$ come from?
- Hall's theorem

Suggested problems from the book: 1C, 1F, 1G, 1I, 1J, 2A, 2B, 2F (note: you have one vertex for each $2 \le i \le m$), 4E, 5A

Computational/application type questions:

- Can there exist a graph G whose vertices have degrees $1,1,2,3,5,8$? If so, can it be simple? If so, assume it is connected. Is it a tree? What about $1,1,2,3,4$? $1,1,1,1,2,2,3,3$?
- Draw a somewhat interesting graph G. Does G admit an Eulerian circuit? Does it admit a Hamiltonian circuit? What is its girth? What is its chromatic polynomial? What is its chromatic number? Put weights on its edges and compute a cheapest spanning tree.
- Draw a somewhat interesting labeled tree T on n vertices. How many automorphisms does it have? How many spanning trees of K_n are isomorphic to T? What is its Prüfer code? For how many other labeled trees T' on n vertices is it the case that $\deg_T(i) = \deg_{T'}(i)$ for each $1 \le i \le n$? What is its chromatic number?
- Write down a plausible Prüfer code for a tree with 10 vertices. To what tree does your Prüfer code correspond?
- Consider a simple graph G with 22 vertices. Write down an expression for the most number of edges G could have without admitting K_5 as a subgraph.