Math 108: Final Review

Format: The exam will be three hours in McCullough 115, from 8:30-11:30 AM on Friday, March 20. It is closed notes. A large chunk of the exam will be computational (e.g., in how many ways can I distribute n identical balls into m distinct boxes?) and applications of theorems and definitions (e.g., inclusion-exclusion). There will be problems that involve counting something two ways, and there will be some relatively friendly proofs. You may be asked to state some of the important theorems. The exam will be cumulative.

Topics:

Graph theory:

- Basic definitions: Graphs, trees, simple graphs, digraphs, components, isomorphisms, automorphisms, valency, subgraphs, paths, walks, spanning trees, bipartite graphs, forests, weighted graphs, vertex colorings, proper colorings, girth, matchings, etc.
- Eulerian and Hamiltonian circuits. Euler's theorem for graphs and digraphs.
- $\sum_{y \in V(G)} \deg(v) = 2|E(G)|$
- Problem 1C: Trees have $n-1$ edges, etc.
- Cayley's theorem and Prüfer codes
- The number of spanning trees on K_n isomorphic to a given T
- The cheapest spanning tree algorithm
- The statement of the four color theorem
- The chromatic polynomial and its meaning, chromatic recurrence: $\chi(G,k) = \chi(G-e,k) - \chi(G \cdot e, k)$
- Basic properties of $\chi(G,k)$, including $\chi(G,k) = k^n - Ek^{n-1} + \dots$, and its relation to $\chi(G)$
- Ramsey's theorem, and the Ramsey numbers $N(p,q) = N(p,q;2)$. You do not need to know about $N(p_1, \dots, p_s; 2)$, $N(p_1, \dots, p_s; t)$, etc.
- Hall's theorem
- The stable matching algorithm

Enumerative aspects:

- Basic counting principles: Binomial and multinomial coefficients, inclusion-exclusion, sticks and dots/stars and bars, etc.
- Derangements: Definition, inclusion-exclusion, the relation to $1/e$
- The Möbius function, $\mu(n)$: Definition, relation to inclusion-exclusion
- Linear recurrence sequences: Basic properties, how to find the generating function, exact formulas
- Catalan numbers: The different problems (walks, parentheses, non-intersecting chords, etc.), the recurrence relation, André reflection, the formula for C_n
- The twelvefold way: Know how to get each entry and what it means in terms of balls and boxes.
- Stirling numbers of the second kind: Definition, recurrence relation, relation to Bell numbers, how to get the inclusion-exclusion formula (13.13)
- Partitions: Recurrence relations for $p_k(n)$, the connection to $a_k(n)$, Ferrers diagrams, generating functions, Euler's theorem on $p_{\text{odd}}(n) = p_{\text{dist}}(n)$, all versions of the pentagonal number theorem

Suggested problems from the book: Homework problems, and 1A, 1D, 1H, 1J, 3C, 3F, 3H, 5Gi, 10C, 10D, 10F (ignore the direct proof), 13C, 13F (first part only), 14B, 14C, 14H, 14L, 15D, 15E, 15H.

A few problems with useful ideas, but content not on the exam: 16B, 16F, 16G (read section for notation), 16H, 17D