Math 250 **The Distribution of Prime Numbers** Fall 2017

Course Information

BLOCK: H+ TTh (Tue, Thu 1:30–2:45)
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OFFICE HOURS: (Spring 2017) Tue 2-3, Wed 10:30-11:30, Fri 1-2
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PREREQUISITES: Math 135 (Real Analysis I) and Math 145 (Abstract Algebra I). Math 63 (Number Theory) and Math 158 (Complex Variables) will be useful, but are not required. In particular, a crash course in the necessary techniques from complex analysis will be given.

TEXT: None.

COURSE DESCRIPTION: The field of analytic number theory began with Euler's observation that the zeta function $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$ "knows" something about the set of prime numbers; in particular, the fact that the harmonic series diverges provides an alternative proof of the infinitude of the primes. These ideas took off when Riemann observed that a sufficient understanding of the complex analytic properties of $\zeta(s)$ would lead to a proof of the so-called *prime number theorem*, the statement that the number of primes up to some number x is about $x/\log x$. A proof of the prime number theorem along the lines that Riemann imagined was found by Hadamard and de la Vallée Poussin at the end of the 19th century, and our understanding of the primes has only grown since. A standard first course in analytic number theory would develop these ideas carefully and would dive into the technical analytic details with gusto.

This is not that course.

Instead, the mantra will be, "How should we *really* think about prime numbers?" Thus, our initial focus will be on the intuition and ideas leading to the proof of the prime number theorem, and we will blackbox the analytic machinery that often obfuscates the actual point of the proof. We will then prove Dirichlet's theorem (how many primes up to x are congruent to $a \pmod q$?), and then see how knowledge of the primes in arithmetic progressions percolates into an understanding of even more subtle properties of primes. Some things we will talk about, related to recent developments in number theory, include:

- The ternary Goldbach problem, that every odd integer is a sum of three primes
- The 2013-14 work of Zhang, Maynard, and Tao on "bounded gaps between primes," our most tangible progress toward the twin prime conjecture
- The Maier matrix method, a tool that can be used, e.g., to show that there are a million primes in a row ending in the digits 8675309

Grades will be based on homework, and a final presentation if enrollment permits.